Elena Simona Lohan

Multipath Delay Estimators for Fading Channels with Applications in CDMA Receivers and Mobile Positioning

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Thesis for the degree of Doctor of Technology to be presented with due permission for public examination and criticism in Tietotalo Building, Auditorium TB109, at Tampere University of Technology, on the 24th of October 2003, at 12 o’clock noon.

Tampere 2003
ABSTRACT

CDMA is the multiple access technique selected for the 3G mobile communications systems and it has a significant role in the research beyond 3G systems. CDMA systems over wireless channels have to cope with fading multipath propagation, which makes the channel estimation an important issue in CDMA receivers. Despite a significant amount of scientific literature on CDMA receivers, there are still open problems regarding the multipath delay and coefficient estimation in hostile environments and the design of low-complexity DSP-based channel estimators for CDMA applications. Good multipath delay estimation techniques can also find their applicability in mobile phone positioning, which is an area with many challenging questions. Additionally, theoretical measures of performance in CDMA detection in the presence of fading multipath channels have mainly been derived for ideal channel estimators. However, developing such analytical models in the presence of channel estimation errors can decrease significantly the computational time when analyzing the performance of different algorithms.

The research results presented in this thesis are focused on three main axes: channel estimation algorithms for low-complexity Rake receivers (with the main focus on the delay estimation part), code synchronization schemes for closely spaced multipath scenarios (including the applications in WCDMA positioning), and analytical studies of the performance of Rake receivers in the presence of multipath fading channels and delay estimation errors. First, the algorithms for the estimation of the main parameters of a fading channel are described, focusing on the channel complex coefficients and the multipath delays. The estimation of some other channel parameters (e.g., Doppler spread, noise variance) is briefly overviewed and a new channel estimation filter with adaptive filter length and fixed coefficients is derived. The problematics of closely spaced paths are emphasized and several solutions are proposed to deal with this situation. Second, two practical applications of the aforementioned channel estimation techniques are shown. The first one is a low-complexity Rake receiver based on interpolation, with incorporated channel estimation block. The second application is the mobile positioning, for which purpose several link-level solutions for the estimation of the delay of the first arriving path are presented. Third, we discuss the main techniques for the theoretical computation of the bit error probability (BEP) of a Rake receiver in the presence of multipath fading channels. The effects of the channel estimation errors on BEP analysis are also discussed. The last part of this thesis is a collection of nine publications that contain the main results of the author’s research work. New algorithms and architectures for the multipath delay estimation in fading channels are introduced and their performance is studied under various CDMA scenarios. A semi-analytical method for computing BEP of a Rake receiver in the presence of code synchronization errors is also presented.
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Preface

The work for this thesis has been carried out during the years 1999-2003 at the Institute of Communications Engineering (ICE), former Telecommunications Laboratory, of Tampere University of Technology, Finland as part of the wider research projects “Digital and Analogue Techniques in Flexible Receivers”, “Advanced Transceiver Architectures and Implementations for Wireless Communications”, and “Advanced Techniques for Mobile Positioning”.

First and foremost I wish to express my deep gratitude to my supervisor, Professor Markku Renfors, for his enthusiastic encouragement, valuable guidance and constant support within the years, and for giving me the opportunity to work on a stimulating research topic.

I would like to thank Professor Markku Juntti from the Telecommunications Laboratory, University of Oulu and Professor Risto Wichman from the Signal Processing Laboratory, Helsinki University of Technology, the reviewers of this thesis, for their constructive feedback and helpful comments.

For four years I had the privilege of being a postgraduate student in the Graduate School in Electronics, Telecommunications, and Automation (GETA), which partially funded my studies and which is gratefully acknowledged. This thesis was also supported by the National Technology Agency of Finland (Tekes) and by Nokia Foundation, which are appreciatively thanked. I also wish to express my gratitude to Professor Iiro Hartimo, the Director of GETA, and to Marja Leppäharju, secretary-coordinator of GETA, for organizing enjoyable and interesting courses and seminars, and for their encouragements and recommendations during the annual GETA meetings.

Distinguished thanks are due to my co-authors, Dr. Ridha Hamila, Abdelmonaem Lakhzouri, Anna Zhuang, Babak Soltanian, and Vesa Lehtinen for numerous hours of working together and for fruitful technical discussions. I express my appreciation to all my present and former colleagues at ICE for creating a relaxed and pleasant working atmosphere. I thank as well many of the colleagues from other institutes of the Department of Information Technology, who shared
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Warm thanks are due to Tarja Erälaukko, Sari Kinnari, Elina Orava, and Ulla Siltaloppi for their always kind help with practical matters. I am thankful to Danny Donoghue for revising the language of the main parts of the manuscript. I want to thank Dr. Adriana Vasilache and Dr. Mihai Enescu for their help with \LaTeX{} during the writing of my thesis. I also express my thanks to the system administrators of ICE and to Florin Lohan for help with Linux and for having so much patience with all computer-related problems during the course of this work.

I would also like to thank the people who, within the years of my thesis, promptly answered my email questions regarding CDMA receivers, in particular Professor René Landry from Québec University, Professor Joseph Cavallaro from Rice University, Timo Eriksson from Nokia Networks, Dr. Mihai Enescu and Marius Sarbu from Helsinki University of Technology, Peter Schulz-Rittich from Aachen Institute for Integrated Signal Processing Systems, Dr. Laurent Schumacher and Benny Vejlgaard from Aalborg University.

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Elena Simona Lohan

September, 2003, Tampere, Finland
List of Publications


[P9] E.S. Lohan and M. Renfors. Performance Analysis of the Rake Receiver in the Presence of Multipath Delay Estimation Errors and Rician Fading Channels. Accepted for publication in European Transactions on Telecommunications.
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<th>Description</th>
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<tr>
<td>3G/4G</td>
<td>Third/Fourth Generation wireless systems</td>
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<tr>
<td>3GPP</td>
<td>3rd Generation Partnership Project</td>
</tr>
<tr>
<td>ACF</td>
<td>Auto-Correlation Function</td>
</tr>
<tr>
<td>AFD</td>
<td>Average Fade Duration</td>
</tr>
<tr>
<td>AOA</td>
<td>Angle Of Arrival</td>
</tr>
<tr>
<td>AWGN</td>
<td>Additive White Gaussian Noise</td>
</tr>
<tr>
<td>BEP</td>
<td>Bit Error Probability (given by theoretical analysis)</td>
</tr>
<tr>
<td>BER</td>
<td>Bit Error Ratio (given by simulation results)</td>
</tr>
<tr>
<td>BPSK</td>
<td>Binary Phase Shift Keying</td>
</tr>
<tr>
<td>BS, BSs</td>
<td>Base Station, Base Stations</td>
</tr>
<tr>
<td>CDMA</td>
<td>Code Division Multiple Access</td>
</tr>
<tr>
<td>CIR</td>
<td>Channel Impulse Response</td>
</tr>
<tr>
<td>CPICH</td>
<td>Common Pilot Channel</td>
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<tr>
<td>CRB</td>
<td>Cramer Rao Bound</td>
</tr>
<tr>
<td>CTF</td>
<td>Channel Transfer Function</td>
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<tr>
<td>DA</td>
<td>Data Aided</td>
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<tr>
<td>DD</td>
<td>Decision Directed</td>
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<tr>
<td>DDPS</td>
<td>Delay-Doppler Power Spectrum</td>
</tr>
<tr>
<td>DF</td>
<td>Decision Feedback</td>
</tr>
<tr>
<td>DFALP</td>
<td>Decision Feedback with Adaptive Linear Prediction</td>
</tr>
<tr>
<td>DLL</td>
<td>Delay Locked Loop</td>
</tr>
<tr>
<td>DPCCH</td>
<td>Dedicated Physical Control Channel</td>
</tr>
<tr>
<td>DPCH</td>
<td>Dedicated Physical Channel</td>
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<tr>
<td>DPDCH</td>
<td>Dedicated Physical Data Channel</td>
</tr>
<tr>
<td>DPS</td>
<td>Doppler Power Spectrum</td>
</tr>
<tr>
<td>DS-CDMA</td>
<td>Direct Sequence Code Division Multiple Access</td>
</tr>
<tr>
<td>DSP</td>
<td>Digital Signal Processing</td>
</tr>
<tr>
<td>EK</td>
<td>Extended Kalman Filtering</td>
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<tr>
<td>EM</td>
<td>Expectation-Maximization</td>
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<tr>
<td>Eq.</td>
<td>Equation</td>
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<td>Eqs.</td>
<td>Equations</td>
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<td>Fig., Figs.</td>
<td>Figure, Figures</td>
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<td>Acronym</td>
<td>Description</td>
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</tr>
<tr>
<td>FIR</td>
<td>Finite Impulse Response</td>
</tr>
<tr>
<td>GA</td>
<td>Gaussian Approximation</td>
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<tr>
<td>GPS</td>
<td>Global Positioning System</td>
</tr>
<tr>
<td>GSM</td>
<td>Global System for Mobile communications</td>
</tr>
<tr>
<td>IC</td>
<td>Interference Cancellation</td>
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<tr>
<td>ICI</td>
<td>Inter-Chip Interference</td>
</tr>
<tr>
<td>IIR</td>
<td>Infinite Impulse Response</td>
</tr>
<tr>
<td>IM</td>
<td>Interference Minimization</td>
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<tr>
<td>IPDL</td>
<td>Idle Period Down Link</td>
</tr>
<tr>
<td>IPI</td>
<td>Inter-Path Interference</td>
</tr>
<tr>
<td>iPS</td>
<td>Improved Pulse Subtraction</td>
</tr>
<tr>
<td>IS-95</td>
<td>Interim Standard-95</td>
</tr>
<tr>
<td>ISI</td>
<td>Inter-Symbol Interference</td>
</tr>
<tr>
<td>LCR</td>
<td>Level Crossing Rate</td>
</tr>
<tr>
<td>LOS</td>
<td>Line Of Sight</td>
</tr>
<tr>
<td>LP</td>
<td>Linear Prediction</td>
</tr>
<tr>
<td>LS</td>
<td>Least Squares</td>
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<tr>
<td>MA</td>
<td>Moving Average</td>
</tr>
<tr>
<td>MAI</td>
<td>Multiple Access Interference</td>
</tr>
<tr>
<td>MEDLL</td>
<td>Multipath Estimating Delay Locked Loop</td>
</tr>
<tr>
<td>MGF</td>
<td>Moment Generating Function</td>
</tr>
<tr>
<td>MIMO</td>
<td>Multiple Inputs Multiple Outputs</td>
</tr>
<tr>
<td>ML</td>
<td>Maximum Likelihood</td>
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<tr>
<td>MLSD</td>
<td>Maximum Likelihood Sequence Detector</td>
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<tr>
<td>MMSE</td>
<td>Minimum Mean Square Error</td>
</tr>
<tr>
<td>MOE</td>
<td>Minimum Output Energy</td>
</tr>
<tr>
<td>MRC</td>
<td>Maximum Ratio Combiner</td>
</tr>
<tr>
<td>MS</td>
<td>Mobile Station</td>
</tr>
<tr>
<td>MSE</td>
<td>Mean Square Error</td>
</tr>
<tr>
<td>MUSIC</td>
<td>Multiple Signal Classification</td>
</tr>
<tr>
<td>MV</td>
<td>Minimum Variance</td>
</tr>
<tr>
<td>NDA</td>
<td>Non-Data Aided</td>
</tr>
<tr>
<td>NLOS</td>
<td>Non-Line Of Sight</td>
</tr>
<tr>
<td>PA</td>
<td>Pilot Aided</td>
</tr>
<tr>
<td>PADD</td>
<td>Pilot Aided Decision Directed</td>
</tr>
<tr>
<td>PDF</td>
<td>Probability Distribution Function</td>
</tr>
<tr>
<td>POCS</td>
<td>Projection Onto Convex Sets</td>
</tr>
<tr>
<td>PN</td>
<td>Pseudorandom Noise</td>
</tr>
<tr>
<td>PS</td>
<td>Pulse Subtraction</td>
</tr>
<tr>
<td>PSK</td>
<td>Phase Shift Keying</td>
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<tr>
<td>QP</td>
<td>Quadratic Program</td>
</tr>
<tr>
<td>Abbreviation</td>
<td>Description</td>
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<tr>
<td>QPSK</td>
<td>Quadrature Phase Shift Keying</td>
</tr>
<tr>
<td>RDLL</td>
<td>Rake Delay Locked Loop</td>
</tr>
<tr>
<td>RRC</td>
<td>(Square) Root Raised Cosine</td>
</tr>
<tr>
<td>SCCL</td>
<td>Sample-Correlate-Choose-Largest synchronizer</td>
</tr>
<tr>
<td>SGA</td>
<td>Standard Gaussian Approximation</td>
</tr>
<tr>
<td>SMET</td>
<td>Successive Multipath Estimation Technique</td>
</tr>
<tr>
<td>SNR</td>
<td>Signal-to-Noise Ratio</td>
</tr>
<tr>
<td>TDOA</td>
<td>Time Difference Of Arrival</td>
</tr>
<tr>
<td>TK</td>
<td>Teager-Kaiser</td>
</tr>
<tr>
<td>TOA</td>
<td>Time Of Arrival</td>
</tr>
<tr>
<td>UMTS</td>
<td>Universal Mobile Telecommunication System</td>
</tr>
<tr>
<td>WCDMA</td>
<td>Wideband Code Division Multiple Access</td>
</tr>
<tr>
<td>WMSA</td>
<td>Weighted Multi-Slot Averaging</td>
</tr>
<tr>
<td>WSS</td>
<td>Wide Sense Stationary</td>
</tr>
<tr>
<td>WSSUS</td>
<td>Wide Sense Stationary with Uncorrelated Scattering</td>
</tr>
</tbody>
</table>
a_l channel path amplitudes
(e.g., a_{l,v} is the l-th path amplitude of the v-th user)
b square of the data symbol envelopes
(for constant envelope modulations)
b^2(n) the n-th complex data symbol
B bilinear form
c speed of light (3 \times 10^8 \text{ m/s})
c^n(n) chips of the PN sequence
(e.g., c^n_k is the k-th chip of the n-th symbol)
E_b bit energy
E_s symbol energy
(E_s = bE_b for constant envelope modulations)
f frequency
f_D maximum Doppler spread
g(\cdot) pulse shaping function, after the matched filtering
g_{1}(\cdot) reference code pulse shaping function
g_{R}(\cdot) receiver matched filter function
(i.e., matched to the transmitter pulse shape)
g_{T}(\cdot) transmitter pulse shaping function,
h(\cdot) channel impulse response
H(\cdot) frequency response
(of a filter or of a channel)
i, i_1 indices without fixed meaning, (e.g., used in multiple sums, or
to define matrix elements, or as sample indices)
I_0(\cdot) the zero-th order modified Bessel function of the first kind
I^n_u sum of all interference factors affecting n-th symbol of u-th user
j the imaginary unit (j = \sqrt{-1})
J(\cdot) various cost functions (e.g., J_{\text{Wiener,MSE}}(\cdot))
J_0(\cdot) the zero-th order Bessel function of the first kind
k, k_1 chip indices
k_{NDA} phase ambiguity factor in NDA estimation
K_l Rician factors
l, l_1 channel paths’ indices
L number of channel paths
L_r number of fingers used in a Rake receiver
M modulation order (i.e., modulation alphabet size)
n, m symbol indices
N number of symbols during the processing time interval
N_0 noise power spectral density
N_{BS} number of Base Stations (when there is
distinction between ‘users’ and ‘BSs’)
N_{NC} non-coherent integration length
\( N_{pf} \)  
filter length (for the fine channel coefficient estimation filter)

\( N_f \)  
number of future symbols

\( N_p \)  
number of past symbols

\( N_s \)  
number of samples per chip (oversampling factor)

\( N_u, N_v \)  
number of users

\( p(\cdot) \)  
probability distribution function

\( P_l \)  
average path powers

\( P_a \)  
average bit error probability

\( P_b \)  
instantaneous bit error probability

\( r(iT_s) \)  
received signal (discrete-time model)

\( r(t) \)  
received signal (continuous-time model)

\( S_F \)  
spreading factors

\( t \)  
time

\( T \)  
symbol period

\( T_c \)  
chip period

\( T_s \)  
sampling period

\( u, v \)  
indices denoting users

\( \text{var}(\cdot) \)  
variance operator

\( w \)  
Wiener filter coefficients

\( y \)  
output of a Rake receiver, after the MRC

\( z \)  
output of a despreading Rake finger (before MRC)

\( W, Z \)  
interference terms at the output of a Rake finger

\( \alpha_l \)  
complex channel coefficients

\( \tilde{\alpha}_l, \bar{\alpha}_l \)  
coarse estimates of the complex channel coefficients

\( \hat{\alpha}_l \)  
fine estimates of the complex channel coefficients

\( \delta(\cdot) \)  
Dirac pulse

\( \delta_K(\cdot) \)  
Kronecker delta function

\( (\Delta t)_{coh} \)  
coherence time of the channel

\( \Delta \)  
variation (e.g., \( \Delta t \) is the time variation)

\( \eta(\cdot), \tilde{\eta}(\cdot), \bar{\eta}(\cdot) \)  
noise processes (narrowband or wideband)

\( \gamma(\cdot) \)  
signal-to-noise ratio at the receiver output

\( \Gamma(\cdot) \)  
Euler function

\( i \)  
term denoting a constant factor

\( \kappa \)  
number of correlation branches

(in feedback delay estimators)

\( \lambda \)  
eigenvalues

\( \mu \)  
mean values

\( \nu \)  
mobile speed

\( \Phi_h(\cdot) \)  
channel ACF

\( \Psi_h(\cdot) \)  
channel DDPS
Φₐ(·) channel coefficient ACFs
Ψₐ(·) channel coefficient DPSs
ρ elements of correlation matrices or of correlation vectors
θ correlation vectors
θₐ channel path phases
θ₀₀ initial phases
ψ integration variable
τ delays (used both in continuous-time and discrete-time domains)
\( \hat{\tau}_l \) delay estimates (discrete time)
τₘₐₓ maximum channel delay spread
ξ delay variation index (continuous time)
Υ(·) moment generating functions
Σ signal or channel correlation matrices

Boldface small letters stand for vectors
b vector of data bits
c vector of channel coefficients
r vector of received signal samples
v vector of noise samples
z vector of despreader outputs

Boldface capital letters stand for matrices (with the exception of the expectation operator)
O all-zero matrix
A matrix of bit amplitudes (square-root energies)
B matrix used in the definition of a bilinear form
C channel matrix
E(·) expectation operator
G pulse shape deconvolution matrix
I unity matrix
Q quadratic form matrix
R, R₁ pulse shaping correlation matrices
S signatures matrix
ℂ set of complex numbers
ℝ set of real numbers
* conjugate operator
T transpose operator
H Hermitian operator
≜ equal by definition
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Chapter 1

Introduction

1.1 Background and motivation

Wireless channels pose great challenges as mediums for reliable communications. A signal transmitted via a wireless channel is affected by various types of distortions, such as reflections, diffractions, and scattering [162]. The signal received at any point can be seen as a sum of multiple transmitted signals, each with different delay, phase, and attenuation. These signals may interfere constructively or destructively, and the result will be a combination of large and small magnitudes, depending on time and location. The time-varying fluctuations of the channel path coefficients are grouped under the name of fading and they are highly random in nature [154], [160], [162]. Various statistical channel propagation models have been proposed and studied in the literature [21], [85], [162]. Some authors also make the distinction between small-scale fading (i.e., fluctuations over short periods of time or distance) and large-scale fading (i.e., the path losses due to motion over large areas) [154], [178]. In this thesis, fading refers only to the small-scale fading phenomenon.

The most-used statistical model for the variation of the multipath fading amplitudes is Rician distribution (Rayleigh distribution can be seen as a particular case of Rician distribution) [6], [162], [178], [191]. The random fluctuating phases of the channel paths are usually modeled as uniform distributed and the most encountered channel model is the linear tapped delay line model [5], [6], [9], [25], [91], [121], [138], [160]. These distributions and channel models have been validated in the literature via numerous channel measurements [100], [144], [159], [168], [208].

By channel estimation we understand the estimation of the multipath delays and channel complex coefficients [65], [121] or, equivalently, the estimation of the time-varying channel impulse response. The knowledge about the channel behavior has multiple applications in wireless communications receivers. The estimates
of the path delays are used for synchronization and, when needed, for mobile positioning, while the channel complex coefficient estimates are used in coherent receiver structures, such as the maximum ratio combiner (MRC). The estimates of the channel complex coefficients may also be needed for the signal quality estimation, for power control, for handover, for adaptive rate control, for transmit and receive antenna diversity, and for advanced multiuser receiver architectures [28], [91], [111], [141], [163], [169], [205], [209].

Although communications over fading channels have been studied for more than half a century, there has always been room for new and improved digital signal processing (DSP) solutions for the estimation of fading channel parameters. When designing DSP solutions for mobile communication systems, one has to deal with a multitude of propagation scenarios (e.g., indoors and outdoors, slow and fast fading, etc.) and one has to take into account the omnipresent requirement of high data rates and high mobility. The higher the signal bandwidth, the more likely it is that the channel becomes selective in frequency and the signal spectrum is subjected to different gains and phase shifts across the frequency band [160], [162]. Moreover, increased mobile speeds cause the channel to be fast fading and decrease the time correlation between channel samples, making the channel estimation more difficult.

The keywords when designing a wireless digital receiver are flexibility, reliability, and low complexity for all the receiver building blocks [29], [197], including the channel estimation part. Low-complexity requirements are motivated by economical drivers such as low cost and low power consumption. Good performance in a multitude of scenarios and high flexibility are also desired to meet various manufacturers’ and operators’ specifications and to keep the pace with ever-evolving telecommunication systems. The multiple access technology selected for today’s standards of third generation (3G) wireless communications is the direct sequence code division multiple access (DS-CDMA) [1], [5], [154]. It is also likely that future wireless systems (sometimes referred to as 4G systems) will retain a CDMA component [126]. Therefore, some of the DSP algorithms developed for 3G systems may be adapted and used in the context of future wireless systems as well. The recent European 3G standard, with wideband CDMA (WCDMA) [1] air interface, introduced new concepts such as intercell asynchronous operation and pilot channels associated with individual data channels [5]. These new features (compared, for example, with the narrowband IS-95

\[\text{In this thesis we will use the short notation CDMA, standing for the DS-CDMA technique. However, in a broader sense, the CDMA term is sometimes used in the literature to cover all the spread-spectrum-based multiple access techniques, such as time-hopping, frequency-hopping, and direct sequence [177].}

\[\text{Sometimes, the European 3G systems are also called Universal Mobile Telecommunications Systems (UMTS) [197]. However, since 'universality' of the standard has not yet been established, the acronym WCDMA is preferred instead, referring both to the air interface and to the system as a whole.}\]
CDMA standard [154]) confer several advantages to the system, such as easier continuous deployment from outdoors to indoors, more accurate channel estimation, and coherent diversity reception, but require also new receiver algorithms and solutions. A Rake receiver is the optimum CDMA receiver in a single-user fading channel, provided that accurate channel estimates are available at the receiver [160]. A Rake receiver consists of several despreading units (or fingers), followed by an MRC block. Although Rake receivers proved to be sub-optimal in the presence of multiple access interference (MAI) [91], [121], [200], they are still the main building blocks of today’s CDMA receivers [1], [52]. Most of the channel estimation algorithms proposed in this thesis have been applied and analyzed in the context of Rake receivers, motivated by the general requirements of low-complexity receivers, especially from the mobile terminal point of view. Nevertheless, most of the methods presented here could also be extended to more advanced receiver architectures, such as the receivers based on interference cancellation or on linear equalization [76], [91], [102], [121], [147].

A relatively new and particularly challenging problem in CDMA systems is the situation of closely spaced paths [25], [44], [52], [113], [172], [216], [219], where different replicas of the transmitted signal arrive at the receiver within less than one chip interval. Closely spaced paths are overlapping in time domain and they are usually “seen” at the receiver as a single path. Hence, some diversity is lost. The loss of diversity is particularly significant in channels with low delay spreads (e.g., indoors), where the number of paths is typically low and the marginal multipath diversity gain (i.e., the diversity obtained by increasing by one the number of multipaths) is important. Moreover, detecting the closely spaced path scenarios and being able to estimate the delays of the individual paths is of utmost importance in mobile positioning applications. The mobile position computation is often done based on the delay estimates of the line-of-sight (LOS) between several base stations (BSs) and the mobile receiver [35], [99], [110], [156], [189], [210]. When an LOS component is obstructed by closely spaced non line-of-sight (NLOS) components, the position estimation accuracy can be severely degraded [124], [125], [216], [219].

Theoretical studies of fading channel behavior and of receiver performance deterioration due to imperfect parameter estimates have also gained particular attention during the last decades. Due to the complexity of emerging wireless communication systems, exhaustive theoretical models are still not available in the literature and many simplifying assumptions are usually made when analyzing a wireless system. In WCDMA communications, multiple random parameters, with a priori unknown underlying distributions, should be taken into account, such as the fading channel parameters, the MAI, the pseudorandom-noise (PN) code characteristics, etc. Moreover, when the signal bandwidth is limited, due, for example, to some frequency regulations, the analysis usually becomes highly non-linear (and hence more difficult) because of the pulse-shape waveforms, such
as the widely used square root raised cosine (RRC) waveforms [1]. Many efficient theoretical models for the BEP analysis of CDMA communications are available in the literature under the assumptions of Rayleigh/Rician fading channels, no channel estimation errors, rectangular pulse shapes, and no coding [95], [97], [121], [122], [129], [160], [175], [207]. The analysis becomes more difficult when the errors in the estimation of the channel coefficients are also considered [65], [91], [138], [157], [175]. The impact of delay estimation errors is even more difficult to study via closed form expressions, especially when bandlimiting pulse shapes have to be considered [14], [65], [157], [188]. For CDMA systems with different types of channel coding or antenna diversity, an exact value of the bit error probabilities is usually hard to obtain, and lower or upper bounds on the performance have to be used instead [68], [175], [195].

1.2 State of the art

The optimum receiver in a fading channel, in the sense of minimum sequence error probability, is the maximum likelihood sequence detector (MLSD), which estimates the data and the channel parameters (i.e., delays and complex coefficients) in a joint manner [66], [160], [203]. The complexity of MLSD (exponential with the length of the data sequences and channel impulse responses) is prohibitive for practical applications with wideband signals. Suboptimal solutions which are simpler to implement can be obtained via differentially coherent or non-coherent receivers, or via decoupling the channel estimation and data detection in a coherent receiver. The performance of differentially coherent and noncoherent receivers is usually poorer than that of the coherent receivers [138], [175]. The receivers of choice for the 3G systems are the coherent ones [1], which are the main topic in this dissertation. Coherent receivers need channel estimation [160]. The channel estimation can be further decoupled into channel coefficient estimation and multipath delay estimation, to allow for even less complex implementations [65], [121].

Channel estimation algorithms for CDMA receivers are usually classified into data-aided (DA), decision-directed (DD) and non-data aided (NDA) approaches, according to the situation when data symbols are known, estimated, or unknown by the receiver [91], [121], [138], [142]. The DA case corresponds to the transmission of some known training sequences (or pilot symbols), code-multiplexed [1], [78] or time-multiplexed [1], [5], [9], [74] with the data symbols. The DD approaches can be seen as two-stage structures, where preliminary data decisions are taken to help the channel estimation task and the channel coefficient estimates are used in the data decision process [11], [91], [142]. The NDA approaches (also called blind channel estimators) estimate the channel impulse response without using known data or data decisions. Instead, the statistical properties of the
transmitted signal are exploited [22], [138], [142], [194]. The data modulation is removed using a nonlinear operator, such as squaring or envelope detection.

When studying the different algorithms for the estimation of the channel complex coefficients, most authors usually assume that perfect knowledge about the path delays is available at the receiver [5], [9], [11], [43], [74], [78], [106], [128], [161]. However, in a realistic CDMA receiver both channel coefficient estimation and delay estimation tasks should be performed, either in a joint or in a decoupled manner. Most of the papers describing joint channel estimation solutions employ high-complexity approaches based on maximum-likelihood (ML) theory [20], [30], [49], [142], [181], [214]. Few authors also addressed the problem of both channel coefficient and delay estimation via decoupled channel estimation architectures [65], [121], [217].

The DA and DD channel coefficient estimators have become increasingly popular due to the fact that current CDMA standards specify the use of pilot symbols both in uplink and downlink directions [1], [72], [154]. Typical channel coefficient estimation is done in the narrowband domain, after the despreading operation [9], [11], [74], [91], [128], [161]. Channel coefficient estimation directly from the wideband domain, where the signal-to-noise ratio (SNR) is very low, is theoretically possible, but not well-documented in the literature. Wideband channel coefficient estimates may be needed, for example, in chip equalization-based receiver structures [47], [76], [137]. A straightforward approach in this case is to interpolate between narrowband channel coefficient estimation samples. Another approach, based on adaptive filtering and time-multiplexed pilot symbols, is proposed in [51] and called chip-level adaptive channel estimation. However, blind chip equalizers are usually preferred [40], [76], because channel coefficient estimation is very difficult at low SNRs. The work of this thesis focuses on narrowband channel coefficient estimators.

Delay estimation solutions are mostly based on feedback tracking loops [25], [38], [52], [57], [61], [64], [81], [123], [124], [125], [170], [173], [187]. Several open-loop solutions have also been proposed in the literature, such as those based on ML theory [55], [149], [150], based on deconvolution approaches [44], [67], [112], [113], [216], [219], based on differential correlation [165], or based on subspace approaches [86], [90], [135], [164], [172], [186], but their performance has traditionally been studied in the presence of rectangular pulse shapes and short codes.

Several solutions for solving closely spaced paths can be found in the literature, such as approaches based on delay locked loops (DLL) with interference cancellation or minimization [25], [52], multipath estimating DLL (MEDLL) [149], [150], deconvolution [44], [113], [216], and multiple signal classification (MUC...
SIC) [67], [86], [112], but some of the major questions such as the impact of the bandlimiting pulse shaping waveform, the impact of the code length, and the complexity and performance comparison of the proposed algorithms are usually disregarded.

The theoretical analysis of receiver performance in the presence of channel estimation errors and fading channels has also been developing during the past decades. Performance limits of the estimators such as the Cramer Rao bound (CRB) or the mean square error (MSE) of the estimates have been thoroughly analyzed in mobile and fixed environments under several simplifying assumptions such as the absence of fading, single path channels or piecewise-linear pulse shaping waveform (e.g., the rectangular waveform) [23], [49], [186]. The performance bounds of the channel parameter estimation, such as the CRB, depend on the signal bandwidth, the noise level, the channel delay and Doppler spread, etc. [59], [184]. For the situation where random nuisance parameters are present, various CRB-like bounds have been derived in the literature in order to make the analysis simpler or more tractable. Examples of such bounds include the modified CRB [60], [143], the asymptotic CRB [143], the hybrid CRB [60], the Miller Chang bound [60], [134], and the Bhattacharyya bound [3], [134].

On the other hand, global measures of receiver performance are usually based on BEP. The analytical methods for computing the BEP of CDMA receivers in fading channels can be mainly divided into two categories: those based on the knowledge of the probability distribution function (PDF) or of the moment generating function (MGF) of the received signal-to-noise ratio (e.g., at the output of the MRC) [7], [160], [175], [176], [185] and those based on the statistical properties of a quadratic receiver4 [16], [65], [95], [121], [138]. The semi-analytical approaches based on one of the two above categories have also gained considerable attention during the past years [34], [95]. The target features in modeling the system performance should be low complexity and easy extension of the model to cases with uncertainties in the estimated signal parameters (such as imperfect delay estimation, various multipath profiles with different statistical properties, etc.).

1.3 Scope of the thesis and parallel work

The core of this thesis is the design and analysis of fading channel estimation algorithms suitable for CDMA receivers. The aim of this thesis is to introduce new channel estimation algorithms in order to solve some of the problems related to the propagation of the CDMA signal via fading multipaths. The main focus is on the low-complexity solutions for the Rake receiver, with a particular empha-

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4Most CDMA wireless receivers can be seen as particular cases of the general quadratic receiver introduced by Bello in [18] and later developed by Barrett in [16].
sis on the delay estimation part. Multipath delay estimation in the presence of closely spaced paths is addressed in detail, with the focus on both MRC receivers and mobile positioning applications. Several new algorithms and architectures are proposed in the context of CDMA multipath channel estimation. The results of the proposed algorithms were mainly obtained in a downlink WCDMA environment, built according to the current standard [1]. However, the applicability of the proposed algorithms is not restricted to the downlink WCDMA. Since the focus was on decoupled channel-estimation and decoding structures, the performance measure was mainly the raw bit error ratio (BER), i.e., without coding-decoding stages.

The motivation behind the choice of the simple Rake receiver structure with decoupled channel estimation and decoding blocks came from the low-complexity requirements of today’s manufacturers of CDMA receivers and from the fact that most of the work presented in this thesis has been done as part of implementation-oriented projects. Although the studies of multiuser detection techniques and iterative joint channel estimators and decoders have gained considerable attention in the research community during the last few years [46], [91], [94], [111], [205], the low-cost Rake receivers with decoupled channel estimation and decoding are still those most encountered in practical applications. Recently, more and more research work has also been focused on multiple antenna systems, triggered by the potential ability of the multiple-inputs-multiple-outputs (MIMO) systems to improve the transmission capacity [2], [12], [42], [48], [58], [62], [153], [163], [195]. The analysis of MIMO systems in the context of CDMA receivers is outside the scope of this thesis, due again to the extra complexity of their implementation compared to single antenna systems. However, the assumption of single antenna systems was not a fundamental pre-requisite in developing the delay estimation algorithms proposed in this thesis. The author believes that most of the proposed delay estimation algorithms could be extended to MIMO CDMA systems as well. Good treatments of the problematics and impact of imperfect channel estimation in multiantenna CDMA systems can be found, for example, in [28], [141], [163], [169], [209].

This thesis also aims at providing a general framework for the theoretical analysis of Rake receiver performance in the presence of fading channels and channel estimation errors. Existing approaches are presented and a new method for computing BEP in the presence of multipath delay errors is introduced.

Since the topic of CDMA channel estimation has gained considerable attention especially after the proposal of CDMA as a multiple access technique for 3G systems, it is natural that parallel work has been independently done in other research groups as well. For example, an extension of the pilot aided decision directed (PADD) channel coefficient estimation algorithm proposed and studied by the author and her co-authors, both in the absence and in the presence of delay estimation errors in [223] and [P1], respectively, has recently been presented in
In [158] the path delays were assumed known. A delay estimation approach partially similar to the pulse subtraction (PS) algorithm, which is the core idea of the improved PS of [P3], was described in [140] in the context of uplink WCDMA TDD mode, and called successive multi-path estimation technique (SMET). There, the pulse to be subtracted is the auto-correlation of the RRC waveform (observed in [P3] to give sub-optimal results compared to the ideal triangular pulse) and the method is applied only for distant paths (4 chips apart).

Parallel studies on the delay estimation of the closely spaced paths in WCDMA environments have been performed independently by Fock & al. and a MUSIC-based solution, somehow similar to our solution of [P5], has been proposed in [53]. However, in [53] only the synchronous uplink situation was considered and the effect of the pulse shaping waveform on the algorithm was not discussed. In [53] it is also stated that the MUSIC-based solution exhibits very good results, but no comparison with other methods is given. Our comparison with Teager-Kaiser (TK) estimator in [P5] showed that the MUSIC algorithm is usually outperformed by the TK estimator and that the RRC waveform has an important effect on the performance of both algorithms.

Parallel results regarding LOS estimation in the presence of multipath interference have recently been reported in [87], [88], [202], but the algorithms described there are different from those presented in the publications [P6]–[P8].

### 1.4 Outline and main results of the thesis

The contributions of this thesis are in the area of multipath fading channel estimation for CDMA applications. This thesis consists of an introductory part with seven chapters and a compendium of nine publications ([P1]–[P9]), attached as appendices. The structure of this thesis was chosen with the intention of providing a comprehensive and unified summary of the problems and challenges in channel estimation and of its applications in CDMA fading multipath environments. The parts which are widely covered in the publications [P1]–[P9] are only briefly reviewed in the introductory part of the thesis and the parts which ensure the link between publications are described in more detail. The new algorithms and the main results of the thesis are originally reported in [P1]–[P9]. In the thesis, the emphasis is on a downlink WCDMA system where different users’ dedicated signals are transmitted synchronously within a cell and different BSs transmit asynchronously [1]. Good overviews of the WCDMA standard can be found in [75], [154].

Chapter 2 gives a short overview of the fading channel and signal models. In Chapter 3, the channel coefficient estimation is considered first and then DA, DD, and NDA methods are described. The choice of the filters for accurate channel
coefficient estimation is analyzed and a new method is proposed to compute the optimal filter length of a smoother-type filter with fixed coefficients.

The problem of multipath delay estimation is addressed next and several feed-forward architectures are proposed (both in DA/DD and NDA modes) as a viable alternative to the traditional feedback loop structures. The combined architecture with separate channel coefficient estimation and delay estimation blocks is given in [P1]. Particular attention is paid to the situation with closely spaced paths and several new solutions are given to distinguish between overlapping paths. The first solution, described in detail in [P3], is a low-complexity technique, referred to as the improved pulse subtraction (IPS) method, based on the ML theory and the MSE optimization. The second solution, described in [P4] and, partially, also in [P5] and [P6], is based on the non-linear TK operator and it also has a reduced implementation complexity. The third solution, given in detail in [P5], is an extension of the MUSIC algorithm to the systems with long codes. The main drawback of the MUSIC algorithm is its high complexity. Moreover, it is shown that the performance of the MUSIC algorithm is rather limited in the presence of RRC pulse shaping. The fourth solution, first introduced in [P8], is based on deconvolution theory: a new projection onto convex (POCS) algorithm is proposed and its performance is compared with that of other existing deconvolution methods.

The performance of the proposed algorithms is studied in environments built in accordance with WCDMA specifications, with the focus on applications such as low-complexity Rake receivers and high-accuracy LOS estimation for mobile positioning, as discussed in Chapter 4. Details related to these two applications are also found in [P2] and [P7]. Moreover, to improve the LOS estimation process in the absence of a priori knowledge about the number of channel paths, a new adaptive threshold is introduced, as explained in [P7], and its performance is analyzed for a WCDMA downlink receiver.

Chapter 5 deals with the theoretical modeling of the receiver performance in the presence of multipath fading channel and channel estimation errors (the main emphasis is on the delay estimation errors). The existing analytical methods for BEP computation are summarized and a semi-analytical model for BEP computation is proposed by the author and derived in detail in [P9]. The method of [P9] has the advantage of giving fast and reliable results for Rake receivers in Rician fading channels and incorporates the multipath delay errors and pulse shaping effects. However, its applicability to channel coefficient estimation errors and coded systems (e.g., antenna diversity, channel coding) remains an open issue.

A short summary of the thesis publications [P1]–[P9] is given in Chapter 6, where the author’s contribution to the publications is clarified as well. The general conclusions of the thesis and the remaining open issues are given in Chapter 7. The original results of the thesis, which are summarized in the introductory part, are mainly reported in the publications, attached as appendices to the thesis.
Chapter 2

Channel and signal models

In this chapter we introduce the channel and signal models for CDMA communications over fading channels. We present first the fading channel model used in this thesis and we explain the reason behind the choice of a linear time-variant filter model. The main channel parameters are enumerated and the typical statistical models for their underlying distributions are given. Then, we present the received baseband signal model for a CDMA receiver with maximum ratio combining.

2.1 Fading channel models

Multipath propagation over wireless channels is associated with fading effects which can be directly related to the impulse response of the mobile radio channel [138], [160], [162]. Most frequently, a mobile radio channel is modeled as a linear filter with a time-varying impulse response $h(t, \xi)$ and a finite number of paths [12], [17], [91], [121], [138], [152], [160], [162]:

$$h(t, \xi) = \sum_{l=1}^{L} \alpha_l(t) \delta(\xi - \tau_l(t)), \quad (2.1)$$

where $\alpha_l(t) = a_l(t)e^{j\theta_l(t)}$ is the time-varying complex coefficient of the $l$-th path, $(a_l(t)$ and $\theta_l(t)$ are its amplitude and phase, respectively), $\delta(t)$ is the Dirac pulse, $L$ is the number of channel paths, $\tau_l(t)$ is the time-varying delay of the $l$-th path, $t$ is the time variation (due to the receiver motion in space), and $\xi$ is the delay variation (due to multipath propagation). The channel parameters $a_l(t)$, $\theta_l(t)$, and $\tau_l(t)$ can usually be seen as realizations of some random processes, with underlying PDFs $p_{a_l}(a)$, $p_{\theta_l}(\theta)$, and $p_{\tau_l}(\tau)$, respectively [175]. The channel impulse response (CIR) $h(t, \xi)$ is sometimes called the input delay-spread function [17]. Fig. 2.1 shows an example of a time-varying channel impulse response with Rayleigh distributed amplitudes and 6 channel paths.
The Fourier transform of the CIR is the channel transfer function (CTF):

\[ H(\xi, f) = \int_{-\infty}^{\infty} h(t, \xi) \exp(-j2\pi ft)dt, \]

(2.2)

The CTF is also randomly varying with respect to the frequency and delay axes, as illustrated in Fig. 2.2.

We note that several nonlinear fading channel models have also been proposed in the literature and they usually fall into two categories: the models based on expanding the nonlinear characteristics into a power series [19], [63] and the Markov chain models, where the individual channel states are still characterized by Eq. (2.1) [115], [221].

In this thesis, the discrete linear time-variant model for the channel impulse response is used. The motivation came from the majority of experimental field measurement results reported in the literature so far [4], [100], [144], [159], [208] and from the fact that linear models are much simpler to simulate and analyze than the nonlinear ones. Moreover, the channel modeling with a finite number of taps is more convenient and natural for computer simulations and has been proved to cover a multitude of wireless propagation scenarios [145], [154].

Figure 2.1: Example of a time-varying CIR with finite number of paths.
2.1.1 WSSUS model

In many physical channels the statistics of the fading paths may be assumed approximately stationary for sufficiently long time intervals. It is therefore customary, for the sake of simplified analysis, to make the assumption that we have a wide sense stationary (WSS) channel [17], [85], [162]. We say that the linear time-variant channel is WSS if its channel impulse response has its mean and variance invariant under a translation in time direction [17], [160]:

\[
\begin{align*}
\mathbb{E}(h(t, \xi)) &= \mu_h(\xi) \\
\mathbb{E}(h(t_1, \xi_1)h^*(t_2, \xi_2)) &= \Phi_h(t_1 - t_2, \xi_1, \xi_2).
\end{align*}
\]  

(2.3)

Above, \(\mathbb{E}(\cdot)\) is the expectation operator, \(\mu_h(\xi)\) is the channel mean with respect to the time axis, \(\Phi_h(\cdot)\) is the time-delay channel auto-correlation function (ACF) (which is stationary under a time translation and non-stationary under a delay translation). Moreover, if the channel coefficients are assumed uncorrelated, the channel becomes wide sense stationary with uncorrelated scattering (WSSUS) [17], [85]. This translates into stationarity property both in time and delay directions [17]:

\[
\Phi_h(t_1 - t_2, \xi_1, \xi_2) = \Phi_h(t_1 - t_2, \xi_1 - \xi_2).
\]  

(2.4)

The Fourier transform of the time-delay channel ACF represents the delay-Doppler power spectrum (DDPS) \(\Psi_h(f, \xi)\) of the channel (also called the scatter-...
For WSSUS channels, $\Psi_h(f, \xi)$ is given by \cite{160}

$$
\Psi_h(f, \xi) = \int_{-\infty}^{\infty} \Phi_h(t, \xi) \exp(-j2\pi ft) dt.
$$

Under the WSSUS assumption and the discrete channel tap model, the channel time-delay ACF is related to the ACF of the complex coefficients $\Phi_{\alpha_l}(\cdot)$ via the following relationship \cite{17}:

$$
\Phi_h(\Delta t, \Delta \xi) = \sum_{l=1}^{L} \Phi_{\alpha_l}(\Delta t) \Phi_{\tau_l}(\Delta t, \Delta \xi),
$$

where

\begin{align}
\Phi_{\alpha_l}(\Delta t) &= E(\alpha_l(t)\alpha_l^*(t + \Delta t)) \\
\Phi_{\tau_l}(\Delta t, \Delta \xi) &= E(\delta(\Delta \xi - \tau_l(t))\delta(\Delta \xi - \tau_l(t + \Delta t))).
\end{align}

We note that under the assumption of constant channel tap delays, from Eqs. (2.5) and (2.6) it follows that:

$$
\Psi_h(f, \xi) = \sum_{l=1}^{L} \Psi_{\alpha_l}(f) \delta(\xi - \tau_l),
$$

where $\Psi_{\alpha_l}(\cdot)$ is the Doppler power spectrum (DPS) of the $l$-th path (i.e., the Fourier transform of the channel coefficient ACF $\Phi_{\alpha_l}(\cdot)$). Fig. 2.3 shows two examples of DDPS for a Rayleigh fading channel with maximum Doppler spread $f_D = 506$ Hz and constant versus time-varying channel path delays (for the assumptions about the path delays distributions see Section 2.1.2).

The frequency-dispersive nature of the fading channel is best described by two inter-dependent parameters, namely the Doppler spread and the coherence time. The Doppler spread is a measure of the spectral broadening due to the time rate of change of the channel parameters \cite{162}. If the delay axis $\xi$ is considered fixed (e.g., $\xi = 0$), then the maximum Doppler spread $f_D$ is given by the maximum frequency range over which the DDPS is essentially non-zero \cite{160}. The dual of the Doppler spread is the coherence time of the channel, which represents the maximum time range over which the channel time-delay ACF (at fixed delay $\xi$) is essentially non-zero (e.g., when the channel time-delay ACF decreases at half of its maximum value) \cite{162}. The Doppler spread and the coherence time are inversely proportional. The maximum Doppler spread $f_D$ depends on the mobile speed $\nu$ and the carrier frequency $f_c$ \cite{162}:

$$
f_D = \frac{\nu}{c} f_c,
$$

13
Figure 2.3: Examples of a time-varying DDPS; constant path delays (left plot) and random path delays (right plot).

where \( c = 3 \times 10^8 \) m/s is the speed of light.

The complementary parameters describing the time dispersive nature of the channel are the delay spread and the coherence bandwidth [160], [162], [178], [179]. The channel delay spread can be seen as the maximum delay range over which the channel time-delay ACF (at fixed time \( t \)) is essentially non-zero. The delay spread and the coherence bandwidth are also inversely proportional to one another. The higher the delay spread, the higher time dispersion the channel suffers, and, hence, it becomes more selective in frequency.

2.1.2 Typical distributions

The statistical properties of the channel random parameters are well-studied in the literature [160], [162]. Typically, the channel paths’ time-varying envelopes \( a_l(t) \) are seen as realizations of Rician distributed random processes [4], [6], [174], [191], whose probability distribution functions are given by:

\[
p_{a_l}(a) \triangleq \text{Proba}(a_l(t) = a) = \begin{cases} 
2a(1+K_l)e^{-K_l} \exp\left(-\frac{(1+K_l)a^2}{P_l}\right)I_0\left(2a\sqrt{\frac{K_l(1+K_l)}{P_l}}\right), & \text{if } a \geq 0 \\
0, & \text{otherwise}
\end{cases}
\]

where \( P_l = \text{E}(|a_l(t)|^2) \) is the average power of the \( l \)-th path, \( K_l \) is the Rician factor of the \( l \)-th path, ranging from 0 (Rayleigh) to \( \infty \) (no fading) [174], [175],
and $I_0(\cdot)$ is the zero-th order modified Bessel function of the first kind:

$$I_0(x) \triangleq \frac{1}{2\pi} \int_0^{2\pi} \exp(x \cos \vartheta) d\vartheta = \sum_{i=0}^{+\infty} \frac{x^{2i}}{2^{2i}(i!)^2}$$

(2.10)

Rayleigh fading is a particular case of Rician distribution, with $K_I = 0$. The Rayleigh fading model typically agrees well with experimental data for mobile systems where no LOS path exists between the transmitter and the receiver antennas [100]. The Rician fading model is more general and incorporates also the LOS situations, both for terrestrial and satellite communications [174], [175]. Examples of Rician PDFs are shown in Fig. 2.4.

![The Rician PDF for different values of the Rician factor K](image)

Figure 2.4: Examples of Rician PDFs for three Rician parameters: $K = 0$, $K = 2$ and $K = 10$; normalized average path power $P=1$.

The time-varying phases of the channel paths are mostly modeled as uniform distributed [85], [162]:

$$p_{u_i}(\theta) \triangleq \text{Proba}(\theta_i = \theta) = \begin{cases} \frac{1}{2\pi}, & \text{if } 0 \leq \theta \leq 2\pi \\ 0, & \text{otherwise} \end{cases}$$

(2.11)

The reason behind the uniform distribution of phases comes from the Gaussian assumption of the complex fading random process characterizing each multipath component [162].

The multipath delays are usually assumed constant [52], [113], [121], [124], [150], [186] [216], [219]. For terrestrial communications this is a reasonable assumption, due to the small Doppler shift usually encountered in these systems. If
we consider, for example, the downlink connection of WCDMA [1] and a mobile receiver moving at 80 km/h (with respect to the transmitter), a delay variation of one tenth of a chip\(^1\) is achieved in about 0.35 seconds, meaning that the delay variation due to Doppler shift can be neglected during typical observation times of orders of tens of milliseconds\(^2\). The time-varying delay models are seldom used and mainly in the context of satellite communications, where a linear or quadratic time dependence\(^3\) is typically assumed [56], [167].

For the Rician fading WSSUS channel model (which is the model used within this thesis), the general expression of the channel coefficient ACF is given by [191]:

\[
\Phi_{\alpha l}(t) = \frac{P_l}{K_l + 1} J_0(2\pi f_D t) + \frac{P_l K_l}{K_l + 1} \exp \left( -2\pi f_D t \cos(\theta_{0l}) \right),
\]

(2.12)

where \(\theta_{0l}\) is the angle between the LOS component and the receiver direction of movement and \(J_0(\cdot)\) is the zero-th order Bessel function of the first kind:

\[
J_0(x) \equiv \sum_{i=0}^{\infty} \frac{(-1)^i}{\Gamma(i+1)i!} \left( \frac{x}{2} \right)^{2i},
\]

(2.13)

and \(\Gamma(\cdot)\) is Euler function \(\Gamma(x) = \int_0^{+\infty} \vartheta^{x-1} \exp(-\vartheta) d\vartheta\).

The DPS for the Rician fading coefficient can be expressed in the following form [191]:

\[
\Psi_{\alpha l}(f) = \frac{P_l}{K_l + 1} \frac{1}{\pi \sqrt{1 - \left( \frac{f}{f_D} \right)^2}} + \frac{P_l K_l}{K_l + 1} \delta(2\pi f - \cos(\theta_{0l})).
\]

(2.14)

We mention that for Rayleigh fading \((K_l = 0)\), from Eqs. (2.12) and (2.14) we obtain the well-known Jakes equations [85], [220]. An example of the normalized channel ACF and DPS for a Rayleigh fading channel is shown in Fig. 2.5.

### 2.2 CDMA baseband signal model

In an CDMA multicell multiuser environment with \(M\)-PSK modulation, the received baseband signal via a multipath fading channel with additive white Gaussian noise (AWGN) can be modeled as:

\(^1\)One chip interval in WCDMA is equal to 0.26 \(\mu s\); the chip rate is equal to 3.84 Mcps.

\(^2\)WCDMA physical channels are organized into frames of duration of 10 ms. Each frame is split in 15 slots and each slot corresponds to one power control period [1]. Typical channel estimation times are of the order of slot period.

\(^3\)A linear dependence has the form \(\tau(t) = \iota + \iota_v t\), and a quadratic dependence has the form \(\tau(t) = \iota_v t + \iota_a t^2\), where \(\iota, \iota_v, \) and \(\iota_a\) are constant factors.
Figure 2.5: Ideal ACF and DPS for a Rayleigh fading channel tap.

\[
\begin{align*}
    r(iT_s) &= \sum_{v=1}^{N_u} \sum_{m=-\infty}^{\infty} \sqrt{E_{b_v}^{(m)}} b_v^{(m)} \sum_{k=1}^{S_{F_v}} c_k^{(m)} \sum_{l=1}^{L} \alpha_{l,v}^{(m)} g(iT_s - mT_v - kT_c - r_{l,v}^{(m)}T_s) + \eta(iT_s),
\end{align*}
\]

where \(r(iT_s)\) are the samples of the received signal, \(T_s\) is the sampling interval, \(i\) is the sample index, \(E_{b_v}^{(m)}\) is the transmitted bit energy of the \(m\)-th symbol and the \(v\)-th user (or base station \(^4\)), \(b_v^{(m)}\) is the \(m\)-th transmitted data symbol of the \(v\)-th user \(^5\), \(\alpha_{l,v}^{(m)}\) is the complex fading coefficient of the \(l\)-th path of the \(v\)-th user, corresponding to the \(m\)-th symbol and \(r_{l,v}^{(m)}\) is the delay (expressed in samples) introduced by the \(l\)-th multipath component of the \(v\)-th user, corresponding to the \(m\)-th symbol. Furthermore, \(c_k^{(m)}\) is the code value for the \(k\)-th chip of the \(v\)-th user, during the \(m\)-th symbol. If long codes are employed, the code sequences are different from one symbol to the other. For short codes, the code value \(c_k^{(m)}\) does not depend on the symbol index.

Above, \(T_v\) is the symbol interval of the \(v\)-th user, \(T_c\) is the chip interval (assumed to be the same for all users), \(T_c = T_s N_s\), where \(N_s\) is the number of

\(^4\)In what follows, by ‘user’ we understand any type of signal that is transmitted in the mobile communication system, such as user dedicated channels or BS common channels. We remark that comprehensive descriptions of the signals transmitted in a WCDMA system can be found in [1], [5].

\(^5\)The symbols are modeled as real numbers if binary phase shift keying (BPSK) modulation is used, and as complex numbers if higher order phase shift keying (PSK) modulation is used.
samples per chip (or the oversampling factor), \( g(t) \) is the chip pulse shape after the matched filtering \( (g(t) = g_T(t) \otimes g_R(t), \) \( g_T(\cdot) \) is the transmitter pulse shape, \( g_R(\cdot) \) is the receiver filter matched to the transmitter pulse shape, \( S_{F_v} \) is the spreading factor of the \( v \)-th user \( (S_{F_v} = T_v / T_c) \), and \( L \) is the number of channel paths\(^6\). The code chips are normalized in such a way that \( \sum_{k=1}^{S_{F_v}} |c_{k,v}^{(m)}|^2 = 1, \forall v, m. \)

In Eq. (2.15), \( \eta(iT_s) \) are the samples of a complex AWGN of double-sided power spectral density \( bN_0 \), where \( b = |b^{(m)}_v| \) (since \( M \)-PSK modulation is assumed for all users, the symbols have constant envelope\(^7\)). \( M \)-PSK modulation has been selected according to the 3G standards \([1]\) and to the current standards for CDMA-based navigation systems \([56], [72], [73], [124]\) where BPSK and QPSK modulations are mainly used.

We note that in publication \([P9]\) the same signal model was described in continuous-time domain for the sake of easier theoretical analysis. However, from now on, the discrete-time description of Eq. (2.15) will be preferred, since all the received signal processing (including the matched filtering) is assumed to be done in digital domain.

---

\(^6\)If the users experience different channels with different paths, \( L \) is taken equal to the maximum number of paths that might be encountered and the channel coefficients corresponding to ‘missing’ paths are considered to be zero.

\(^7\)For example, \( b = 1 \) for BPSK modulation, and \( b = 2 \) for quadrature PSK (QPSK) modulation. The symbol energy \( E_s \) is related to the bit-energy \( E_b \) via \( E_s = bE_b \).

---

Figure 2.6: Block diagram of a Rake receiver with \( L_r \) fingers and separate channel estimation blocks.

The simplest coherent receiver for CDMA communications is a Rake receiver, whose block diagram is illustrated in Fig. 2.6. In our architecture of Fig. 2.6, the channel coefficient estimation is done in the narrowband domain and it is therefore decoupled from the delay estimation block. Different techniques may be used
independently for each of these two tasks. In this thesis, we assume that uncoded systems are used; hence, the channel decoding stage illustrated in Fig. 2.6 is not incorporated. If the channel estimation blocks use the data decisions (hard or soft), the estimation is done in a decision directed mode. The output of several Rake fingers in Fig. 2.6 is coherently combined before the data detection. The number of Rake fingers $L_r$ may be different from the number of channel paths $L$, since there is no a priori knowledge about $L$. The reference signal in Fig. 2.6 is the code sequence, possibly multiplied with data symbols (DA case) or with data decisions (DD case).

The despread signal of the $u$-th user, at the output of the $l_1$-th finger during the $n$-th symbol can be written as:

$$z_{l_1,u}^{(n)} = \frac{1}{S_{F_u} N_s} \sum_{i=nS_{F_u} N_s}^{(n+1)S_{F_u} N_s} r(iT_s) \sum_{k_1=1}^{S_{F_u}} (c_{k_1,u}^{(n)})^* g_1(iT_s - nT_u - k_1T_c - \hat{\tau}_{l_1,u}^{(n)} T_s),$$

(2.16)

where $\hat{\tau}_{l_1,u}^{(n)}$ is the estimated delay of the $l_1$-th path of the $u$-th user, during the $n$-th symbol (expressed in samples), and $g_1(\cdot)$ is the pulse shape of the replica code at the receiver. For unshaped replica codes, $g_1(\cdot)$ is the rectangular pulse shape [121]. For shaped replica codes, $g_1(\cdot)$ is different from the rectangular pulse shape (we might have, for example, $g_1 \equiv g$ or $g_1 \equiv g_T$).

It follows from Eqs. (2.15) and (2.16) that each Rake finger output $z_{l_1,u}^{(n)}$ can be written as:

$$z_{l_1,u}^{(n)} = \sqrt{E_{b_u}^{(n)} b_u^{(n)} \alpha_{l_1,u}^{(n)}} \mathcal{R} (\hat{\tau}_{l_1,u}^{(n)} - \tau_{l_1,u}^{(n)}) + \sqrt{E_{b_u}^{(n)} b_u^{(n)}} \sum_{k=1}^{S_{F_u}} L \sum_{l=1}^{L} \sum_{l \neq l_1} \mathcal{Z}_{l,k,k,u,u}^{(n,n)} + \sqrt{E_{b_u}^{(n)} b_u^{(n)}} \sum_{k=1}^{S_{F_u}} \sum_{k_1=1}^{S_{F_u}} \sum_{l=1}^{L} \sum_{k \neq k_1} \mathcal{Z}_{l,k,k_1,u,u}^{(n,n)} + \sum_{m=-\infty}^{\infty} \sum_{m \neq n} \sqrt{E_{b_v}^{(m)} b_v^{(m)}} \sum_{k=1}^{S_{F_v}} \sum_{k_1=1}^{S_{F_v}} \sum_{l=1}^{L} \sum_{l \neq l_1} \mathcal{Z}_{l,k,k_1,u,u}^{(m,n)} + \sum_{v=1}^{N_v} \sum_{m=-\infty}^{\infty} \sum_{v \neq u} \sqrt{E_{b_v}^{(m)} b_v^{(m)}} \sum_{k=1}^{S_{F_v}} \sum_{k_1=1}^{S_{F_v}} \sum_{l=1}^{L} \sum_{l \neq l_1} \mathcal{Z}_{l,k,k_1,v,u}^{(m,n)} + \sum_{k=1}^{S_{F_u}} \mathcal{W}_{k,u}^{(n)}$$

(2.17)
where the notations $Z_{l,k,k_1,v,u}^{(m,n)}$ and $W_{k,u}^{(n)}$ stand for:

$$Z_{l,k,k_1,v,u}^{(m,n)} = c_{k,v}^{(m)} c_{k_1,u}^{(n)} \alpha_{l,v}^{(m)} R \left( nT_u - nT_v + (k_1 - k)T_c + (\hat{\tau}^{(n)}_{l_1,u} - \hat{\tau}^{(m)}_{l_1,v}) T_s \right), \quad (2.18)$$

and, respectively,

$$W_{k,u}^{(n)} = \frac{1}{S_{F_u} N_s} \sum_{i=nS_{F_u} N_s}^{(n+1)S_{F_u} N_s} \eta(iT_s) g_1(iT_s - nT_u - kT_c - \hat{\tau}^{(n)}_{l_1,u}) g_1(iT_s), \quad (2.19)$$

and $R(\cdot)$ is the normalized correlation function between the received signal pulse shape and the reference code pulse shape (normalization means that $R(0) = 1$):

$$R(\tau T_s) = \frac{1}{S_{F_u} N_s} \sum_{i=nS_{F_u} N_s}^{(n+1)S_{F_u} N_s} g(iT_s + \tau T_s) g_1(iT_s). \quad (2.20)$$

The subscript $u$ has been dropped from the correlation function, for clarity reasons, and also because it is reasonable to assume that all the users have the same pulse shape function. We mention that the correlation function $R(\cdot)$ is invariant with respect to symbol index $n$.

The first term in the summation of Eq. (2.17) represents the useful signal component, the second term is due to inter-path interference (IPI), the third term is due to inter-chip interference (ICI), the fourth term is due to inter-symbol interference (ISI), the fifth term is due to the multiple access interference, and the last one is due to the complex additive white Gaussian noise. If IPI, ICI, ISI, MAI and AWGN terms are grouped under the generic noise term notation $\hat{\eta}_{l_1,u}^{(n)}$, then the output of the Rake finger of Eq. (2.17) becomes:

$$z_{l_1,v,u}^{(n)} = \sqrt{E_{b_n} b_{u,n}^{(n)} } \alpha_{l_1,u}^{(n)} R \left( z_{l_1,u}^{(n)} - \tau_{l_1,u}^{(n)} \right) + \hat{\eta}_{l_1,u}^{(n)}. \quad (2.21)$$

If the Gaussian approximation (GA) is used, by virtue of central limit theorem, $\hat{\eta}_{l_1,u}^{(n)}$ term can be seen as a Gaussian noise of zero mean and a certain variance $\sigma^2$, directly related to $N_0$, to the channel multipath profile and to the MAI parameters.

The output of the MRC for the $n$-th symbol of the $u$-th user is

$$y_u^{(n)} = \sum_{l_1=1}^{L_u} \hat{\alpha}_{l_1,u}^{(n)} z_{l_1,u}^{(n)}, \quad (2.22)$$
where \( (\hat{\alpha}_{l,u}^{(n)})^* \) is the conjugate of the estimated complex coefficient of tap \( l \) during the \( n \)-th symbol, and \( L_r \) is the number of Rake fingers used in the MRC.

An equivalent CDMA received signal model is the matrix model of [91], [121]. Since the matrix model will be used in Chapter 5 for BEP analysis of the quadratic receiver, we also review here the matrix notations for the received signal model. We assume that the baseband processing is done in groups of \( N \) symbols and we group the received samples \( r(iT_s), i = 1, \ldots, NSF N_s \) under the vector notation \( r \in \mathbb{C}^{N_u S_F N_s \times 1} \). Here we assumed without loss of generality\(^8 \) that all the users have the same spreading factor \( S_F \). Then, the vector of the received samples is equal to [91], [121]:

\[
    r = SCAb + v, \tag{2.23}
\]

where \( S \) is the signature matrix (including the pulse shaping effect):

\[
    S = \left( s_{1,1}^{(1)}, \ldots, s_{L,1}^{(1)}, \ldots, s_{1,u}^{(n)}, \ldots, s_{1,N_u}^{(N)}, \ldots, s_{L,N_u}^{(N)} \right) \in \mathbb{C}^{N_u S_F N_s \times NN_u L}, \tag{2.24}
\]

with vector elements

\[
    s_{l,u}^{(n)} = \left( 0, \ldots, 0, s_{l,u,n}^{(n)}, 0, \ldots, 0 \right)^T \in \mathbb{C}^{NN_u S_F \times 1}. \tag{2.25}
\]

In the Eq. (2.25) there are \( n N_s S_F + \tau_{l,u}^{(n)} \) zeros at the beginning, corresponding to the \( l \)-th path delay of the \( u \)-th user and the \( n \)-th symbol, and \( (N - n) N_u S_F - \tau_{l,u}^{(n)} \) zeros at the end (for \( n = N \), the sequence is truncated). The elements \( s_{l,u,n}^{(n)} \) represents the signature of the \( u \)-th user, \( l \)-th path and \( n \)-th symbol, corresponding to the sample \( n_s \), \( n_s = 1, 2, \ldots, N_s S_F \) (the pulse shape effect is included in the signatures). The matrix \( C \) in Eq. (2.23) is the channel matrix and it has a block-diagonal structure:

\[
    C = \text{diag} \left( c_{1,1}^{(1)}, \ldots, c_{N_u,1}^{(1)}, \ldots, c_{1,1}^{(N)}, \ldots, c_{N_u,1}^{(N)} \right) \in \mathbb{C}^{NN_u L \times NN_u N}, \tag{2.26}
\]

with vector elements equal to: \( c_{u}^{(n)} = \left( \alpha_{1,u}^{(n)}, \ldots, \alpha_{L,u}^{(n)} \right)^T \in \mathbb{C}^{LN_u \times 1} \).

The matrix \( A \) in Eq. (2.23) is the diagonal bit square-root energy matrix

\[
    A = \text{diag} \left( \sqrt{E_{b_1}^{(1)}}, \ldots, \sqrt{E_{b_{N_u}}^{(1)}}, \ldots, \sqrt{E_{b_1}^{(N)}}, \ldots, \sqrt{E_{b_{N_u}}^{(N)}} \right) \in \mathbb{R}^{N_u N \times N_u N}. \tag{2.27}
\]

The vector \( b \) is the vector of users’ symbols

\[
    b = \left( b_{1,1}^{(1)}, \ldots, b_{N_u,1}^{(1)}, \ldots, b_{1,1}^{(N)}, \ldots, b_{N_u,1}^{(N)} \right)^T \in \mathbb{C}^{N_u N \times 1}. \tag{2.28}
\]

\(^8\)Indeed, spreading one symbol with a code of spreading factor \( S_F \) can be viewed as spreading two identical symbols with different codes of lower spreading factor \( S_F/2 \).
The last term in Eq. (2.23) is the vector of noise samples:

\[ \mathbf{v} = (\eta(T_s), \eta(2T_s), \ldots \eta(N_sS_FT_s))^T \in \mathbb{C}^{N_NS_F \times 1}. \] (2.29)

The despreading operation can be viewed as a multiplication with the hermitian of the signature matrix at the receiver (e.g., simultaneous despreading for all users):

\[ \mathbf{z} = \hat{\mathbf{S}}^H \mathbf{r} = \hat{\mathbf{S}}^H \mathbf{S} \mathbf{C} \mathbf{A} \mathbf{b} + \hat{\mathbf{S}}^H \mathbf{v}, \] (2.30)

where \( \mathbf{z} = \begin{pmatrix} z_{1,1}^{(1)} & \cdots & z_{1,N_u}^{(N)} & \cdots & z_{L,1}^{(1)} & \cdots & z_{L,N_u}^{(N)} \end{pmatrix}^T \in \mathbb{C}^{NN_u L \times 1} \) is the vector of despreading fingers outputs and \( \hat{\mathbf{S}} \) is a matrix with the same structure as matrix \( \mathbf{S} \), where the true channel path delays \( \tau_{l,u}^{(n)} \) were replaced with the estimated path delays \( \hat{\tau}_{l,u}^{(n)} \).

The output of the MRC of Eq. (2.22) can be written in matrix form as well:

\[ y_u^{(n)} = \left( \hat{\mathbf{c}}_u^{(n)} \right)^H \mathbf{z}_u^{(n)}, \] (2.31)

where \( \hat{\mathbf{c}}_u^{(n)} \) is the vector of estimated channel complex coefficients of the \( n \)-th symbol and the \( u \)-th user. For the above equation, the number of Rake fingers \( L_r \) was assumed to be equal to the number of true paths \( L \). However, the model can be straightforward extended to \( L_r \neq L \).
Chapter 3

Channel estimation algorithms

This chapter provides an overview of the fading-channel estimation problematic and main algorithms. The channel coefficient estimation and delay estimation tasks are addressed separately. The main features and the main challenges for each of these approaches are discussed.

3.1 General methods for channel coefficient estimation

In this section, we assume that the multipath delays are correctly estimated at the receiver and we are interested only in the channel complex coefficient estimation part. The problem of multipath delay estimation will be addressed in Section 3.3.

3.1.1 DA and DD methods

The received signal, after the filter matched to the transmitter pulse shape, is simultaneously despread in $L_r$ fingers. The channel estimation is done at symbol level (in narrowband domain), because the low chip energy-to-noise ratio specific to an CDMA system would make a chip-level estimation much more difficult. Moreover, simulation results [5], [9], [223] showed that a symbolwise estimation is sufficient for velocities of the mobile up to 120 km/h, since the channel is not significantly changing during one symbol interval. Under the assumption of correct path delay estimates, it follows from Eq. (2.21) that the output of the $l_1$-th Rake finger during the $n$-th symbol interval is$^1$:

$$
z_{l_1}^{(n)} = \sqrt{E_b^{(n)}b^{(n)}\alpha_{l_1}^{(n)}} + \eta_{l_1}(n). \quad (3.1)$$

If the transmitted symbol $b^{(n)}$ and the corresponding transmit bit energy are known, and the noise level $\eta_{l_1}(n)$ is low compared to the signal level, a coarse estimate of

$^1$The user index $u$ has been dropped for clarity reasons.
the $l_1$-th path of the channel can be obtained through:

$$\hat{\alpha}_{l_1}^{(n)} \approx \frac{\tilde{s}_{l_1}^{(n)} b(n)^*}{b \sqrt{E_b^{(n)}}}$$  \hspace{1cm} (3.2)

where $b \triangleq |b^{(n)}|^2$ is the square of the data symbol envelope.

In a realistic case, the receiver has the prior knowledge of the pilot symbols only. The pilot symbols may be either continuously transmitted (e.g., the Common Pilot Channel CPICH in WCDMA downlink or the Dedicated Physical Control Channel DPCCH in WCDMA uplink [1]) or time-multiplexed with the data symbols (e.g., the Dedicated Physical Control Channel DPCCH in WCDMA downlink [1]).

When the pilot channel is transmitted continuously, the channel coefficient estimates are typically obtained via multiplication with the pilot symbols (see Eq. (3.2)), followed by low-pass filtering to diminish the noise [78], [91], [96], [121], [204]. The choice of the adequate filter length and coefficients is one of the most important tasks in the channel coefficient estimation process, as it will be discussed in Section 3.2. There is a fundamental tradeoff between reducing the noise level and keeping the fading spectrum of the channel coefficients undistorted.

When a continuous pilot channel is available, the best option is an DA channel coefficient estimator. The DA architecture consists basically of a correlator (between the incoming signal and the reference pilot signal), followed by a fine estimation filter. In DA techniques, when the pilots are time-multiplexed with data, the traditional ways to estimate the channel coefficients corresponding to data symbols are either via weighted multi-slot averaging (WMSA) [5], [9], [74] or via interpolation techniques [P2], [9], [105], [121], [128], [198], [204]. The raw channel estimates corresponding to pilot symbols are first obtained using Eq. (3.2). Then, they are either weighted and averaged over several slots and hold constant for each symbol within one slot [5], [9], [74], or interpolated in order to obtain symbolwise estimates of the channel [P2], [105], [128], [198], [204]. The WMSA method exhibits good results in case of slowly fading environments [9]. For faster fading, interpolation methods perform better [9], [204]. The linear interpolation is the best choice in terms of complexity, and its performance is close to that of higher-order interpolation methods [128], [204].

An alternative to interpolation or WMSA-based DA approaches is to use DD methods, where unknown data symbols are replaced by data decisions. Various DD methods have been proposed in the literature, such as the decision feedback with adaptive linear prediction (DFALP) algorithm [11], [91], [96], [130], [222], [223], the decision feedback (DF) algorithm [222], [223], the pilot aided decision directed algorithm [P1], [84], [223]. The DD methods can be mainly classified into two classes:
1. Feedback DD channel coefficient estimators, such as DFALP or DF [11], [91], [96], [130], [222], [223]: previous channel coefficient estimates (or data decisions) are used to update the current channel coefficient estimates. The block diagram of a feedback DD estimator is shown in Fig. 3.1. The channel coefficient estimation block selects either the past estimate $\hat{\alpha}_{l1}(n)$ or the current estimate given by Eq. 3.2, according to a pre-defined criterion. For example, if we assume that $M$-PSK modulation is used, the decision rule can be defined as follows [130], [223]:

$$
\hat{\alpha}_{l1}(n) = \begin{cases} 
\frac{z_l(n)_b^*(n)}{b\sqrt{E_b(n)}}, & \text{if } b^{(n)} \text{ is a pilot symbol}, \\
\frac{z_l(n)_b^*(n)}{b\sqrt{E_b(n)}}, & \text{if } \left| \arg\left(\hat{\alpha}_{l1}(n)\right) - \arg\left(\frac{z_l(n)_b^*(n)}{b\sqrt{E_b(n)}}\right) \right| \leq \frac{\pi}{2M} \\
\hat{\alpha}_{l1}(n), & \text{if } \left| \arg\left(\hat{\alpha}_{l1}(n)\right) - \arg\left(\frac{z_l(n)_b^*(n)}{b\sqrt{E_b(n)}}\right) \right| > \frac{\pi}{2M} 
\end{cases}
$$

and $b^{(n)}$ is not a pilot symbol.

Above, $\hat{b}^{(n)}$ stands for the estimated $n$-th data symbol and $M$ is the modulation order. The estimates $\hat{\alpha}_{l1}(n)$ are obtained via feedback filtering. Examples of feedback filters proposed in the literature are based on linear prediction (LP) with constant [222], [223] or adaptive coefficients [11], [130], [222], [223].

![Figure 3.1: Feedback DD complex-tap estimation principle.](image)

2. Feedforward DD channel coefficient estimators, such as PADD [P1], [84], [223]: only the current data decisions are used in the channel complex coefficient estimation process, as illustrated in Fig. 3.2. The first stage of a feedforward DD channel coefficient estimator is similar to the DA channel coefficient estimator for time-multiplexed pilot symbols. Based on the
available pilot symbols and on averaging and interpolation methods, preliminary channel coefficient estimates can be obtained as seen in Fig. 3.2.

In the second stage, data decisions are formed at the output of the MRC, and the data modulation is removed from the incoming sequences \( z_1^{(n)}, \ldots, z_{L_r}^{(n)} \) for each Rake finger. An infinite impulse response (IIR) or a finite impulse response (FIR)-type smoother can be used to reduce the noise effect and to obtain the final complex tap estimates \( \hat{\alpha}_1^{(n)}, \ldots, \hat{\alpha}_{L_r}^{(n)} \) (see Section 3.2 for the discussion about the choice of the fine estimation filter).

![Figure 3.2: Feedforward DD complex-tap estimation principle.](image)

A detailed comparison between different DD channel coefficient estimation methods in the absence of code synchronization errors was shown in [223] for a WCDMA downlink scenario with time-multiplexed pilot symbols. Fig. 3.3 shows an example of the BER performance of different DD methods for indoor channel A and vehicular channel B [1]. The impact of the delay estimation errors on the performance of different DA and DD channel coefficient estimation algorithms was studied in [P1].

In practical applications when the pilots are time-multiplexed with the data symbols, combined DA/DD approaches such as PADD algorithm are usually the best choice, both from the complexity point of view and from their robustness to delay estimation errors, as shown in [P1].

The time-multiplexed pilot-based channel coefficient estimators might seem somehow obsolete in WCDMA applications nowadays since the introduction of continuous (code-multiplexed) pilot sequences both in uplink and downlink directions [1], [161]. However, there are still various situations, such as transmit antenna diversity\(^2\) [209] or lack of power control for the common CPICH channels\(^3\) [41], when a time-multiplexed pilot sequence might be still the best available option for coherent detection. Moreover, combined channel coefficient estimators...

---

\(^2\)For example, the CPICH channels transmitted via 2 antennas have orthogonal patterns and some de-patterning process should be performed before they can be used for channel estimation.

\(^3\)CPICH channels are not power controlled, which causes them to be received with very low power by the mobile receivers situated at the cell edges [1].
based on both time-multiplexed and continuous pilots may be used to increased the receiver performance [41], [105], [161].

3.1.2 NDA methods

Non-data aided (or blind) methods may be used when none or very few pilot symbols are available at the receiver. It is usually assumed that the receiver knows the PN code sequence. NDA algorithms in the context of fading channel coefficient estimation and CDMA receivers have not received much attention in the literature, due to several open issues which are generally associated with the blind phase estimators, such as the phase-ambiguity (or cycle-slip) problem [89], [138], and the typical non-coherent loss\(^4\). The phase ambiguity problem can be solved either by using differential or non-coherent detection or by having separate pilot symbols as a phase reference [138]. We will briefly explain here the concepts of NDA channel coefficient estimation when \(M\)-PSK modulation and coherent detection are used (e.g., such as in WCDMA systems [1]).

The despreader output of Eq. (3.1) can be written as a function of the amplitudes and phases of the channel and data symbols:

\[
    z_l^{(n)} = \sqrt{bE_b^{(n)}} a_l^{(n)} e^{\left(j\frac{2\pi m^{(n)} + \theta_l^{(n)}}{M}\right)} + \eta_l^{(n)},
\]

where \(m^{(n)}\) is a random index defining the phase of the \(n\)-th \(M\)-PSK modulated data symbol, taking values in the set \(\{0, 1, \ldots, M - 1\}\). In the absence of noise,

\(^4\)When differentially coherent or non-coherent detection is used instead of coherent detection, the receiver performance is usually deteriorating with up to 3 dB [160].
the $M$-th power non-linearity removes completely the phase of the modulation data symbol:

$$
(z_l^{(n)})^M = \left(\sqrt{bE_b^{(n)} a_l^{(n)}} \right)^M e^{(jM\theta_l^{(n)})}.
$$

(3.4)

Taking the argument of both terms of Eq. (3.4) we obtain an estimate for the channel phase in the form

$$
\tilde{\theta}_l^{(n)} = \frac{1}{M} \arg \left( z_l^{(n)} \right)^M + \frac{k_{\text{NDA}} \pi}{M},
$$

(3.5)

where $k_{\text{NDA}} \in \{-(M-1), \ldots, -1, 1, \ldots, M-1\}$ is the phase ambiguity factor and it is due to the non-uniqueness of the solution of Eq. (3.4). In the presence of noise, some non-coherent averaging over $N_{\text{NC}}$ symbols may be used to reduce the effect of noise, and the coarse estimates for the $l$-th channel-path phase and amplitude are obtained as follows [142]

$$
\begin{align*}
\tilde{\theta}_l^{(n)} &= \frac{1}{M} \arg \left( \frac{1}{N_{\text{NC}}} \sum_{j=n-N_{\text{NC}}}^{n} (z_j^{(j)})^M \right) + \frac{k_{\text{NDA}} \pi}{M}, \\
\tilde{a}_l^{(n)} &= \sqrt{\frac{1}{N_{\text{NC}}} \sum_{j=n-N_{\text{NC}}}^{n} |z_j^{(j)}|^2}.
\end{align*}
$$

The block diagram of an NDA channel coefficient estimator (when the estimation is done in narrowband domain) is shown in Fig. 3.4. Similarly to the DA and DD estimators, fine channel coefficient estimates can be obtained via a supplementary filter (denoted here by fine estimation filter), applied on real and imaginary parts of the estimates. If pilot symbols are available, then the phase ambiguity problem can be removed by disallowing the estimated phases to lie outside a certain interval, which is computed with the help of the pilots [31], [138].

![Figure 3.4: NDA complex tap estimation principle.](image)

Alternatively to the above presented structure, NDA channel coefficient estimates may also be obtained via subspace-based methods [30], [141], [193],
[206]. In [206] it was shown that a subspace-based approach is significantly outperformed by a decision-directed channel estimation. The system considered in [206] was the synchronous downlink IS-95 system. The study of the performance of subspace-based approaches in the context of channel coefficient estimation and realistic CDMA scenarios (such as those in conformity with 3G standards) has not gained much attention in the research literature, due to their high complexity and low robustness to noise. The subspace-based approaches will also be discussed in the context of CDMA delay estimation, in Section 3.3.4.4.

3.2 Fine estimation filter

As seen in the block diagrams of Figs. 3.1, 3.2, and 3.4, the coarse channel coefficient estimates $\tilde{\alpha}_{l1}^{(n)}$ are passed via an IIR or FIR filter (referred to as fine estimation filter), in order to reduce the noise effect and to obtain the final estimates $\hat{\alpha}_{l1}^{(n)}$. The finite impulse response filters are sometimes more adequate for low-complexity receivers than infinite impulse response filters [93] and they will be used in what follows. The channel estimation filter may have either fixed or adaptive coefficients.

Under the assumption of unbiased estimates, the coarse channel coefficient estimates $\tilde{\alpha}_{l1}^{(n)}$ can be re-written as the sum between the true channel coefficients and a noise process $\eta_l^{(n)}$, of zero mean and variance $\sigma^2$.

$$\tilde{\alpha}_{l1}^{(n)} = \alpha_{l1}^{(n)} + \eta_l^{(n)}.$$ (3.6)

The noise variance $\sigma^2$ is usually estimated from the received signal samples [45], [50]. In the next sections, two types of FIR channel coefficient estimation filters are discussed and compared: the classical Wiener filter (with adaptive coefficients) and the smoother filter with fixed coefficients.

3.2.1 Wiener filter

The optimum adaptive filter in the minimum-mean-square-error (MMSE) sense is the Wiener filter [37], [71], [108]. If we select an FIR implementation with $N_{pf_l}$ coefficients, the fine channel-coefficient estimates are obtained as:

$$\hat{\alpha}_{l1}^{(n)} = \sum_{i = -N_{pf_l}}^{N_{f_l}} w_l^{(n+i)} \tilde{\alpha}_{l1}^{(n+i)} = w_l^{(n)} H_l^{(n)}, \quad l = 1, \ldots, L_r,$$ (3.7)

where $N_{pf_l} = N_p + N_f + 1$ is the filter length, $N_p$ is the number of past symbols used by Wiener filter applied to the $l$-th path, and $N_f$ is the number of future symbols (optimally, $N_p$ and $N_f$ tend to infinity, but in practical implementations, they
should be truncated to some finite values [71]). The notation \( w_i^{(n)} \) stands for the vector of the Wiener filter coefficients of the \( l \)-th path, centered at the \( n \)-th symbol: \( w_i^{(n)} = [u_i^{(n-N_p l)} \ldots u_i^{(n+N_f l)}]_T \), and \( \tilde{c}_i^{(n)} \) is the vector of coarse estimates of the channel coefficients \( \tilde{c}_i^{(n)} = [\tilde{\alpha}_i^{(n-N_p l)} \ldots \tilde{\alpha}_i^{(n+N_f l)}]_T \).

The optimal Wiener filter coefficients are found by minimizing the mean square error function \( J_{\text{Wiener}, \text{MSE}}(\cdot) \) with respect to \( w_i^{(n)} \):

\[
J_{\text{Wiener}, \text{MSE}}(w_i^{(n)}) \triangleq \mathbb{E} \left| w_i^{(n)} H c_i^{(n)} - \alpha_i^{(n)} \right|^2. \tag{3.8}
\]

It can be shown that the coefficients which minimize the MSE error of Eq. (3.8) are given by the Wiener-Hopf equation [71]:

\[
w_i^{(n), \text{opt}} = \left( \mathbb{E}(\tilde{c}_i^{(n)} H) \right)^{-1} \mathbb{E}(\alpha_i^{(n)} \tilde{c}_i^{(n)H}) \triangleq \left( \Sigma_{\tilde{c}_i^{(n)} \tilde{c}_i^{(n)H}} \right)^{-1} \theta_{\alpha_i^{(n)} \tilde{c}_i^{(n)}}, \tag{3.9}
\]

where \( \Sigma_{\tilde{c}_i^{(n)} \tilde{c}_i^{(n)H}} \triangleq \mathbb{E}(\tilde{c}_i^{(n)} \tilde{c}_i^{(n)H}) \) is the auto-correlation matrix of the coarse channel coefficient estimation vector \( \tilde{c}_i^{(n)} \) and \( \theta_{\alpha_i^{(n)} \tilde{c}_i^{(n)}} \triangleq \mathbb{E}(\alpha_i^{(n)} \tilde{c}_i^{(n)H}) \) is the cross-correlation vector between the coarse channel coefficient estimation vector and the true channel coefficient.

Under the WSSUS assumption of the channel, the matrix \( \Sigma_{\tilde{c}_i^{(n)} \tilde{c}_i^{(n)H}} \) and the vector \( \theta_{\alpha_i^{(n)} \tilde{c}_i^{(n)}} \) (and hence the Wiener filter coefficients) are independent on the symbol index and their elements can be re-written as:

\[
\rho_{xx,l}(i, i_1) = \Phi_{\alpha_l}(i - i_1) + \sigma^2 \delta_{K}(i - i_1), \forall i, i_1 = 1, 2, \ldots, N_{f_l} + N_{p_l} + 1, \tag{3.10}
\]

and, respectively:

\[
\rho_{\alpha x,l}(i) = \Phi_{\alpha_l}(i - N_{p_l} - 1), \forall i = 1, 2, \ldots, N_{f_l} + N_{p_l} + 1, \tag{3.11}
\]

where \( \rho_{xx,l}(i, i_1) \) is the element corresponding to the \( i \)-th row and the \( i_1 \)-th column of the matrix \( \Sigma_{\tilde{c}_i^{(n)} \tilde{c}_i^{(n)H}} \), \( \rho_{\alpha x,l}(i) \) is the element corresponding to the \( i \)-th row of the column vector \( \theta_{\alpha_i^{(n)} \tilde{c}_i^{(n)}} \), and \( \delta_{K}(\cdot) \) is the Kronecker delta function. We note that for Rayleigh and Rician fading channels, the expression of \( \Phi_{\alpha_l}(\cdot) \) is given by Eq. (2.12).

The coefficients of the Wiener filter depend on the filter length \( N_{p_l} \), on the channel ACF function \( \Phi_{\alpha_l}(\cdot) \), and on the noise variance at the output of the coarse estimation unit \( \sigma^2 \). In practice, these parameters have to be estimated.

From Eqs. (3.8) and (3.9) the MMSE for the optimal Wiener coefficients is:

\[
J_{\text{Wiener,MMSE}}(N_{p_l}, N_{f_l}) = \Phi_{\alpha_l}(0) - \sigma^2 \delta_{K} \left( \Sigma_{\tilde{c}_i^{(n)} \tilde{c}_i^{(n)H}} \right)^{-1} \theta_{\alpha_i^{(n)} \tilde{c}_i^{(n)}}. \tag{3.12}
\]
3.2.2 Filters with fixed coefficients and suboptimal filter length

For practical implementations, filters with fixed coefficients, such as the moving average (MA) filter, are usually preferred to the Wiener filter due to their lower complexity. The only design parameter of such a filter with fixed coefficients is its length, which is either assumed constant [9], [161], [223], or taken equal to the coherence time of the channel (but not adapted to the noise level) [43]. Another approximation based on maximizing the signal power at the output of the matched filter is given in [219], where the optimal MA filter length \( N_{l}^{(opt)} \) for the \( l \)-th path is the solution of the following equation:

\[
\sum_{i=1}^{N_{l}^{(opt)}-1} i \Phi_{\alpha_l}(i) = 0. \tag{3.13}
\]

3.2.3 Filters with fixed coefficients and optimal filter length

Adapting the filter coefficients in an optimal manner is usually quite computationally expensive due to the required matrix inversion of Eq. (3.9). Therefore, we introduce here a smoother with fixed coefficients and adaptive filter length as an alternative to Wiener filter for lower-complexity implementations. We then show the MSE comparison between the proposed filter, the Wiener filter, and the suboptimal filters of Section 3.2.2.

The output of the smoother with fixed coefficients is given by the average of the coarse channel estimates

\[
\hat{\alpha}_l(n) = \frac{1}{N_{pf_l}} \sum_{i=-N_{pf_l}}^{N_{fl}} \tilde{\alpha}_l(n+i). \tag{3.14}
\]

We mention that an MA filter using only the past \( N_{pf_l} \) symbols is a particular case of the smoother described above, with \( N_{fl} = -1 \).

The smoother filter with fixed coefficients is a linear-phase non-causal filter with the transfer function:

\[
H_{smf}(e^{j\omega}) = \frac{\sin \left( \frac{N_{pf_l} \omega}{2} \right)}{\sin \left( \frac{\omega}{2} \right)} \exp \left( j\omega \left( \frac{N_{fl} - N_{pf_l}}{2} \right) \right), \tag{3.15}
\]

introducing a constant delay \( \tau_d \) (positive or negative value):

\[
\tau_d = \frac{N_{fl} - N_{pf_l}}{2}. \tag{3.16}
\]

Since the filter coefficients are fixed, a new criterion can be derived to find the optimal filter length \( N_{pf_l} \) (and the optimal division between the past and future...
symbols). The new criterion is based on minimizing the following cost function with respect to \( N_{p_l} \) and \( N_{f_l} \):

\[
J_{smf}(N_{p_l}, N_{f_l}) = E\left|\hat{\alpha}_l(n-\tau_d) - \alpha_l(n)\right|^2.
\] (3.17)

After straightforward manipulations\(^5\) and using the WSSUS assumption, it can be shown that:

\[
J_{smf}(N_{p_l}, N_{f_l}) = \Phi_{\alpha_l}(0) N_{pf_l} + \Phi_{\alpha_l}(0) + \frac{2}{N_{pf_l}} \left( N_{pf_l} - i \right) \Phi_{\alpha_l}(i) - \frac{2}{N_{pf_l}} \sum_{i=-N_{pf_l}}^{N_{pf_l}} \Phi_{\alpha_l}\left( i - \frac{N_{f_l} - N_p}{2} \right) + \frac{\sigma^2}{N_{pf_l}}
\] (3.19)

Eq. (3.19) can be easily minimized numerically, and the optimal pair \((N_{p_l}^{(opt)}, N_{f_l}^{(opt)})\) can be derived.

Similarly to the Wiener filter coefficients, the optimal pair is dependent on the channel path ACF function and on the noise variance \( \sigma^2 \) at the output of the raw estimation unit. The MMSE cost function is \( J_{smf}(N_{p_l}^{(opt)}, N_{f_l}^{(opt)}) \).

Fig. 3.5 shows the comparison between the MMSE of the channel coefficient estimates when the following filters are used:

- **Fixed length**, i.e., MA filter with fixed coefficients and fixed length.
- **Yousef & al.**, i.e., MA filter with fixed coefficients and filter length given in Eq. (3.13) [219].
- **Wiener, truncated**, i.e., smoother with adaptive coefficients, given by Eq. (3.9), and length truncated at \( N_{pf_l}^{(opt)} \) or at \( N_{pf_l}^{(opt)}/2 \).
- **Coh**, i.e., MA filter with fixed coefficients and filter length equal to the coherence time of the channel [43].
- **Opt MA**, i.e., MA filter with fixed coefficients and optimal filter length, given by Eq. (3.19). We note that the simulation results showed that there were less than 0.5 dB difference if we used a smoother-type instead of an MA filter (when optimal filter lengths are used for both). For clarity purpose, we plotted only the MA-type filter.

\(^5\)We used the fact that

\[
\sum_{i=-N_{pf_l}}^{N_{pf_l}} \sum_{i=-N_{pf_l}}^{N_{pf_l}} \Phi_{\alpha_l}(i - i) = \sum_{i=1}^{N_{pf_l}-1} 2(N_{pf_l} - i) \Phi_{\alpha_l}(i) + N_{pf_l} \Phi_{\alpha_l}(0).
\] (3.18)
The legend in Fig. 3.5 is ordered according to the performance at 20 km/h (decreasing order). In Fig. 3.5, the $SNR = E_b/\sigma^2$ is the bit-energy-to-noise ratio at the output of the despreading unit (we focused only on one path). From Fig. 3.5 we notice that Wiener filter performance degrades very fast when the filter length is decreased. For equal filter lengths (chosen in an optimal manner), the performance of Opt MA filter is very close to the performance of Wiener filter. As expected, using a Fixed length, a Coh, or a Yousef & al. filter is sub-optimal, in the sense that the performance is deteriorating at too low or too high mobile speeds, depending on the noise level. The reason comes from the fact that the length of these three last mentioned filters is not adapted to the noise level, but only to the fading ACF.

3.2.4 Notes on the estimation of mobile speed and noise variance

When deriving the optimal filter lengths or coefficients, the knowledge of the channel ACF and noise variance $\sigma^2$ is needed. For Rayleigh fading channels, the channel ACF is completely determined by the maximum channel Doppler spread, which, in turn, is dependent on the mobile speed and carrier frequency. The channel coefficient estimation problem is therefore closely related to the estimation of the mobile speed and signal-to-noise ratios at different stages in the system. Estimates of the mobile speed and SNR may also be needed for other receiver blocks, such as the power control or the handover decision blocks [37], [108], [191].

The mobile speed estimation algorithms existing in the literature can be briefly grouped in two categories:

1. Time-domain approaches, e.g., based on the covariance approximations, or based on the envelope level crossing rates (LCR) or average fade duration
2. Frequency-domain approaches: based on the Doppler spectrum or on some parametric spectral analysis [109], [191].

The noise variance estimators are usually based on the received-signal second-order moment [45], [50], [79], [218].

3.3 Multipath delay estimation problem

This section gives a brief overview of the multipath delay estimation challenges and main methods in CDMA systems. A detailed framework of different algorithms and their performance in multipath fading channels can be found in [P6]. Furthermore, publications [P3]–[P5] and [P8] describe several new delay estimation algorithms which are only briefly overviewed in this section.

3.3.1 Acquisition and tracking

The multipath delay estimation problem (or, equivalently, the code synchronization problem in spread-spectrum systems [61]) has traditionally been seen as a two-step process:

1. Acquisition stage: the received signal and the locally generated codes are brought into phase with a residual error of a fractional part of a chip. The search space is usually very large (e.g., without a priori information, all the possible phases of the PN code should be searched). In WCDMA systems for example, the search space is divided into several regions, with the help of several synchronization channels [1], [104]. The acquisition issues in CDMA environments have comprehensively been treated in [61], [80], [103], [177].

2. Tracking stage: once the received signal and its replica code are aligned within less than one chip error, fine delay estimates are further obtained via a tracking unit [25], [38], [61], [125], [170], [173], [177].

However, some authors also preferred to treat these tasks jointly, assuming implicitly that the code search space is reduced via, for example, some network-assisted information (e.g., synchronization channels, knowledge of previous estimates, etc.). In this situation, the delay estimation problem can be seen as a tracking problem (i.e., very accurate delay estimates are desired) with initial code mis-alignment of several chips or symbols [44], [53], [55], [69], [86], [90], [217]. This is also the definition of delay estimation that is used in this thesis.
Similarly to the channel coefficient estimators, the delay estimators can be classified into coherent (DA and DD) and non-coherent (NDA) estimators. Furthermore, they can also be divided into feedback-loop delay estimators and feed-forward delay estimators. The delay-estimation types are discussed in sections 3.3.3 and 3.3.4, after defining, in section 3.3.2, the problem of closely spaced multipath components, which is one of the core problems addressed in this thesis.

### 3.3.2 Closely spaced path problem

In multipath propagation it may happen that the first ray is succeeded by one or several multipath components which arrive at the receiver within less than one-chip interval. This situation is found in the literature under the name of *closely spaced-paths* or *overlapping-paths* situation [25], [44], [52], [113], [149], [172], [216], [219]. A closely spaced-path situation is illustrated in Fig. 3.6 for two pulse shapes (the RRC pulse shape with rolloff factor 0.22, similar to [1], and the theoretical rectangular pulse shape). The two channel paths are in phase and have 1 dB difference in their average amplitudes. The spacing between the two paths is 0.75 $T_c$. The receiver 'sees' a combination of the two paths, and the individual effect of each path is not easily distinguishable, especially in the case of the RRC pulses.

![Figure 3.6: Illustration of the closely spaced path situation in the presence of RRC and rectangular pulse shapes.](image)

Traditional correlation-based techniques have a resolution limited by the autocorrelation function width (i.e., to one chip interval) [160]. However, some diversity gain may be achieved in the Rake receiver if closely spaced paths are also incorporated in the MRC [106]. Moreover, being able to separate the first arriving path of the successive interfering paths is of utmost important in mobile positioning applications [69], [87], [119], [124], [125], [149], [218], [219].
3.3.3 Feedback loop delay estimators

3.3.3.1 DLL-based structures

Classically, the multipath delay estimation block is implemented via a feedback loop. The most common feedback structures for the delay estimation are the so-called delay locked loops [25], [52], [57], [61], [101], [121], [155], [170], [187].

Non-coherent DLLs use non-linear devices in order to remove the effect of data modulation. Standard non-coherent (or NDA) DLLs seem to be adequate only in single user, single path channels [123]. Coherent DLLs do not suffer from squaring losses and gain imbalances [57], [121]. However, in spread-spectrum applications, the chip-energy-to-noise ratio is typically too low to obtain the carrier recovery before despreading. Therefore, some “quasi-coherent” (also referred to as coherent in what follows) solutions have been proposed [123], where the phase is corrected after despreading, in an DA or DD mode. Since some signal power will be lost, the solution is in fact a “quasi-coherent” one. When the code tracking loop is implemented in baseband, it is sometimes referred to as digital DLL [57], [187].

Usually, an DLL is based on the correlation between the incoming signal with delayed and advanced versions of a local reference signal [61]. More advanced approaches may use several delayed and advanced versions of the signal, e.g., the very early-very late loop described in [15] for modernized GPS and Galileo applications. An alternative to the DLL structures is the sample-correlate-choose-largest (SCCL) symbol synchronizer, initially proposed in [38], and later studied also in [101]. The main difference between SCCL and DLL is in the way of constructing the error signal. The DLL error signal is based on differences between the outputs of early and late branches. SCCL uses a comparator instead of a subtraction block. The outputs of the early, late, and in-phase correlations are compared and the error signal is built according the position of the maximum between the three correlations.

Another way to mitigate the multipath effect is to use a Rake DLL (RDLL) [124], [173]. A key feature of this tracking loop is the use of a separate multipath channel estimation unit which provides the estimates of the interfering path parameters. The estimated parameters are used in a Rake-like structure to resolve and combine the received multipath components. The RDLL is also conceptually close to the DLL with interference-cancellation (IC) approaches [25], [52]. The DLL with IC subtracts the estimated contribution of interfering paths from the output of the finger tracking the path of interest.

Another improved variant of DLL is the so-called DLL with interference-minimization (IM) technique, proposed in [13], [52]. The idea of an DLL with IM is to filter the outputs of the correlators with some adaptive FIR filter, whose coefficients are designed in such a way to minimize the multipath interference. Again, the knowledge about the interfering path parameters should be obtained.
via an additional multipath channel estimation unit.

A generic block diagram for all the mentioned DLL-based feedback loops is shown in Fig. 3.7. Each branch represents a correlator with a certain code phase (typically, the number of branches is $\kappa = 3$). The output of each correlator is multiplied with the estimated data symbols and channel complex coefficients (when available, e.g., in DA and DD cases) or a non-linear device, such as the envelope detection, is applied for the NDA estimators. The effect of the interfering paths is diminished via IC or IM techniques, using the estimates of a multipath channel estimation unit. The combining rule refers to the way of forming the error signal. The loop filter reduces the noise level and the error signal controls the phase of the local code generator via a numerically controlled oscillator. The output of the mid branch in Fig. 3.7 gives the estimated multipath delay.

![Figure 3.7: DLL-based delay estimators - generic block diagram.](image)

The performance of an DLL is well-characterized by the so-called S-curve, which presents the expected value of the error signal (at the output of the combining rule of Fig. 3.7) as a function of the reference parameter error (i.e., the code mismatch) [P6], [61], [177]. The mathematical definition of the S-curve for the classical DLLs$^6$ can be found in [P6]. The intuition behind the S-curve is

$^6$By classical DLL we understand the feedback structures with three branches (in-phase, early,
the fact that searching for maxima in the correlation function is equivalent with finding the zeros of the gradient, which can be approximated by first-order differences. Therefore, the presence of a multipath component is signalled by the zero-crossings from below\(^7\) of the S-curve.

Two examples of the S-curve behavior of coherent and non-coherent DLLs in the presence of distant and closely spaced paths are shown in Fig. 3.8 and Fig. 3.9, respectively. Both situations with rectangular and RRC pulse shapes are shown in these figures. For both figures a two path fading channel was assumed, with average tap powers 0 and \(-1\) dB, and with delay spacing equal to \(3.25\ T_c\) for Fig. 3.8 and to \(0.75\ T_c\) for Fig. 3.9. Clearly, coherent DLLs perform better than non-coherent DLLs, especially for the RRC cases, when the spurious sidelobes of the correlation functions may be quite high. When the channel paths are sufficiently apart (e.g., Fig. 3.8), the zero-crossings from below of the S-curve show correctly the presence of a multipath component (possibly with some spurious extra zero-crossings from below due to the side lobes cross-correlation in the non-coherent approaches, as seen in the left plot of Fig. 3.8).

![S-curves for coherent and non-coherent classical DLLs in the presence of distant paths. RRC pulse shaping (left plot) and rectangular pulse shaping (right plot).](image)

When the paths are closely spaced, we cannot distinguish any more the correct path delay directly from the output of the S-curve (see Fig. 3.9). In this case, further processing should be done, such as the IC and IM solutions proposed in [13], [25], [52], [64]. The main properties and performance of DLL with IC and late), with error signal formed as the difference between early and late branches, and without any interference cancellation block [61]. The SCCL, the RDLL, the very early-very late DLL, and the DLLs with IC and IM do not belong to this definition.

\(^7\)Equivalently, the zero-crossings from above are sometimes used [69], but we keep here the definition of the S-curve of [P6].

38
were shown in [52] to be quite similar to those of DLL with IM. Therefore, we illustrate in Fig. 3.10 only the case of an DLL with IC. The same channel profile with closely spaced paths of Fig. 3.9 is used. The path of interest is assumed to be the first one, and two situations are plotted: the case with ideal channel estimation (i.e., known channel coefficient and delay of the interfering path) and the case with imperfect channel estimation (known channel coefficient, but estimated delay of the interfering path with a delay error of 0.5 $T_c$). Under the ideal channel estimation assumption, the DLL with IC is able to detect the correct path delay, as seen in Fig. 3.10. However, under imperfect channel estimation assumption, the performance of the DLL with IC starts to degrade rapidly.

Figure 3.9: S-curves for coherent and non-coherent classical DLLs in the presence of closely spaced paths. RRC pulse shaping (left plot) and rectangular pulse shaping (right plot).

Therefore, the main drawbacks of the DLL-based techniques include their reduced ability to deal with closely spaced path scenarios under realistic assumptions (such as the presence of errors in the channel estimation process), their relatively slow convergence [61], and the possibility to loose the lock (i.e., start to estimate the delays with high estimation error) due to the feedback error propagation.

3.3.3.2 EKF-based structures

A different feedback structure for the delay estimation is based on the extended Kalman filter (EKF) and it was originally proposed by Iltis [81] for joint delay and multipath coefficients estimation in single-user CDMA systems with uniformly spaced path delays. Later on, EKF estimators have been extended to the multiuser CDMA environments [127], to the joint channel parameters and Doppler shift estimation [32], and to the fading channels with non-uniform tap delay spacing.
The EKF approach was shown in [118], [119] to provide accurate delay estimates in the presence of closely spaced paths and to converge much faster than the DLLs to the correct solution. The performance of EKF and some typical simulation results in the presence of overlapping paths were also discussed in [P6].

Due to the complexity [173], to the linearization errors [81], [82] and to the high sensitivity of the EKF algorithm to the initialization conditions, such as the error covariance matrices [118], the use of EKF-based delay estimators is not widespread in the research community nowadays. More recently, in the context of CDMA channel estimation, the EKF algorithm has been proposed to be used together with sequential importance sampling or particle-filtering techniques [82], [107], in order to alleviate some of the problems due to the state-space nonlinearities and the initialization of the noise-covariance matrix.

### 3.3.4 Feedforward delay estimators

In the next subsections we give a short overview of the main existing feedforward delay estimation techniques and of the new algorithms introduced by the author in publications [P3]–[P6] and [P8]. The feedforward approaches can also be implemented in an DA, DD, or NDA mode [P1], [132], similar to the channel coefficient estimation.

The generic principle of a feedforward delay estimator is illustrated in Fig. 3.11. The delay estimation can be performed either in the narrowband domain (after despreading or correlation with the reference signal), or in the wideband domain (e.g., via eigenvalue decomposition of the received signal covariance matrix). The reference signal is the pseudorandom code (NDA approaches), multi-
plied with known or estimated data if DA or DD approaches are used. After the correlation or eigenvalue analysis, further optional processing may be done, such as deconvolution, non-linear Teager-Kaiser processing, etc., to improve the delay estimation process in the presence of closely spaced paths. Before a decision is made about the significant path delays, non-coherent block averaging may be used to reduce the effect of noise.

![Diagram](image)

Figure 3.11: Feedforward delay estimators - generic block diagram.

### 3.3.4.1 Maximum-likelihood estimation

Most of the correlation-based solutions are derived from the ML theory [142]. Therefore, ML is a rather generic term used for a variety of feedback and feedforward estimators. However, in a strict sense, ML estimation refers to the joint estimation of unknown channel parameters (e.g., multipath delays and complex coefficients) via the joint minimization of the log-likelihood function $L_{ML}(\cdot)$:

$$L_{ML}(\cdot) \triangleq \ln \left( p_{ML}(r(iT_s)|\tau_1,\ldots,\tau_L,\alpha_1,\ldots,\alpha_L) \right), \quad (3.20)$$

where $p_{ML}(\cdot)$ is the PDF of the received signal, conditional to the channel parameters. Typically, $p_{ML}(\cdot)$ is a multivariate complex Gaussian distribution and the minima of the expression in Eq. (3.20) are obtained via heavy matrix computations [49], [181]. Therefore, ML estimation has prohibitive complexity for practical applications. The large dimensional ML estimation problem can be decomposed into sub-problems of smaller dimensions. Examples of such approaches are the pulse-subtraction techniques, described in Section 3.3.4.2 and the expectation-maximization (EM) algorithm [49], [64], [83]. The EM algorithm was shown to be equivalent to an envelope detection with parallel-interference-cancellation technique [83]. The pulse-subtraction algorithms are also based on the interference cancellation as it will be explained below, hence they can conceptually be seen as a particular form of the EM algorithm.
3.3.4.2 Pulse-subtraction techniques

The class of pulse-subtraction algorithms refers to the algorithms which try to cancel the multipath interference by subtracting a certain reference pulse from the correlation function [P3], [P6], [69], [149], [150], [181], [182]. The pulse is usually built as a function of the pulse-shaping ACF [182], [149] or of the ideal triangular ACF [P3], [P6], [69], and uses the previous estimated path coefficients and delays to cancel the multipath interference. The PS algorithms are therefore implemented iteratively and, according to the way the multipath interference is canceled (i.e., successively or simultaneously), they can be viewed as successive or parallel interference cancellation methods.

An improved pulse-subtraction algorithm is described in [P3] in order to deal with the situation of closely spaced paths and RRC pulse shaping. The main idea there is to build several sets of candidates for the path delay estimates and to choose the sequence which minimize a certain performance criterion, such as the MSE. The iPS algorithm outperforms the PS algorithm in closely spaced-path scenarios.

The performance of pulse-subtraction techniques in the presence of closely spaced paths remains however quite limited compared with some other superresolution approaches [P6] and their main advantage comes from a reduced complexity and straightforward implementation.

3.3.4.3 Deconvolution methods

The expression of the output of the receiver correlators (or fingers) given by Eq. (2.21) can be re-written as [P6]:

\[
\mathbf{z}(n) = \mathbf{\zeta}(n) \mathbf{Gc}(n) + \mathbf{v}(n),
\]

where \(\mathbf{z}(n) \in \mathbb{C}(\hat{\tau}_{\text{max}}+1)\times1\) is a vector whose elements \(\mathbf{z}_l(n)\) are equal to the correlation between the received signal and the reference code, shifted with different time lags \(l\) between 0 and the estimated maximum channel delay spread \(\hat{\tau}_{\text{max}}\) (given in samples\(^8\)), \(\mathbf{\zeta}(n) = \sqrt{E_b(n)b(n)}\) is a symbol-dependent factor, \(\mathbf{G} \in \mathbb{C}(\hat{\tau}_{\text{max}}+1)\times(\hat{\tau}_{\text{max}}+1)\) is the pulse-shape deconvolution matrix, whose elements are \(G_{i,i_1} = R((i - i_1)T_s), i,i_1 = 0,\ldots,\hat{\tau}_{\text{max}}, \mathbf{v}_\eta\) is the vector of noise samples \([\eta_0(n),\ldots,\eta_{\hat{\tau}_{\text{max}}(n)}]^T\), and \(\mathbf{c}(n) \in \mathbb{C}(\hat{\tau}_{\text{max}}+1)\times1\) is the vector of channel coefficients, with elements \(\mathbf{c}_i(n)\):

\[
\mathbf{c}_i(n) = \begin{cases} 
\alpha_i(n) & \text{if a channel path is present at the time delay } iT_s \\
0 & \text{otherwise}
\end{cases}
\]  

\(^8\)For example, \(\hat{\tau}_{\text{max}}\) may span over couple of symbol intervals, according to the uncertainty search region for the multipath delays. If no \textit{a priori} knowledge exists about the search region, \(\hat{\tau}_{\text{max}}\) should cover the whole code period.

42
We remark that slightly different notations compared to [P6] were used in Eq. (3.21) in order to keep a unitary mathematical framework in the introductory part of the thesis.

Resolving multipath components refers to the problem of estimating the non-zero elements of the vector $\mathbf{c}^{(n)}$. Eq. (3.21) illustrates a standard deconvolution problem with unknown vector $\mathbf{c}^{(n)}$. Therefore, the multipath delay estimation problem can be viewed as a standard deconvolution problem. We note that the data modulation effect, included in $\zeta^{(n)}$ factor, should be removed in an DA, DD, or NDA mode before further processing.

Various solutions for this deconvolution problem exist [44], [67], [113], [114], [139], [219]. The main solutions based on deconvolution methods are comprehensively described in [P6] and [P8]. The least squares (LS) solution is shown to fail completely in the presence of RRC waveforms [P8] due to the noise enhancement when the matrix inversion is performed. Constrained or iterative solutions are shown in [P6] and [P8] to be more robust with respect to the pulse-shape waveform choice. An example of such a constrained deconvolution technique is the projection onto convex sets, initially proposed in [114] for solving closely spaced echos in frequency-hopping signals and later applied to CDMA systems in [44], [113], [114]. A novel POCS-based solution (with additional constraints) is described in [P8] in order to increase the resolution of the first arriving-path delay estimation in severe multipath and multicell interference.

A special class of the deconvolution algorithms is also the class of the so-called minimum-variance (MV) estimators [87], [121], [202], where the solution of the Eq. (3.21) is found in frequency domain. It was shown in [121] that the MV solution can be seen as a generalization of subspace-based solutions, in the sense that instead of using a subset of the eigenvalues and eigenvectors of the covariance matrices, the MV estimators take advantage of the full correlation space. The complexity of MV algorithms is quite high and their applicability in CDMA receivers remains rather limited. We note that the minimum output energy (MOE) detector introduced in [166], [190] for multipath delay estimation is also a form of the MV estimation.

From the point of view of the delay estimation accuracy in CDMA environments, the constraint deconvolution methods (such as the POCS variants) are one of the best existing delay estimation methods nowadays [P6], [P8].

### 3.3.4.4 Subspace-based algorithms

The subspace-based algorithms involve the decomposition of the correlation space into a noise subspace and a signal subspace. The subspace-based algorithms involve eigenvector decomposition of high-order matrices, and they remain usually very complex for practical applications. They have traditionally been reported as
methods of high resolution\textsuperscript{9} in parameter estimation problems and near-far resis-
tant in CDMA related estimation problems [86], [186].

In [P5] a MUSIC-based delay estimation method was proposed and described
in detail and its performance in a synchronous single cell downlink WCDMA sys-
tem was asserted via simulations. In [P6], the same MUSIC-based algorithm was
tested in asynchronous multicell downlink WCDMA scenario (with synchronous
intracell interference, in conformity with [1]). It was shown in [P5] and [P6]
that the celebrated high resolution of MUSIC algorithm is usually valid only for
rectangular pulse shapes and it is outperformed in many situations by the lower-
complexity Teager-Kaiser algorithm. Therefore, the subspace-based solutions
seem to be more interesting from the theoretical point of view (e.g., correlation-
space analysis, benchmarks for comparison, etc.), rather than from their practical-
application point of view.

3.3.4.5 Quadratic-optimization algorithms

The delay estimation problem can also be viewed as a quadratic-optimization
problem as shown in [P6]. The original idea belongs to Fuchs [54], [55] and was
applied in a generic narrowband communications system. The quadratic program
(QP) was reformulated for an CDMA system in [P6]. The performance com-
parison with other delay estimation algorithms in [P6] showed that the quadratic-
optimization solution does not provide very good results and has the drawback of a
rather high complexity. The search for the QP solutions can be minimized if a pri-
ori knowledge about the paths delays exists (e.g., delays are previously acquired
within few chips error) and further refinements on the algorithm implementation
are also possible. However, the QP solution has gained but little attention in the
research community.

3.3.4.6 Teager-Kaiser algorithm

The nonlinear quadratic TK operator was first introduced for measuring the real
physical energy of a system [98]. Since its introduction, it has widely been used in
various speech processing and image processing applications and, more recently,
it has also been applied in CDMA applications [P4], [P6], [69], [70]. The discrete-
time TK operator of a complex valued signal \( z(n) \) is:

\[
\Psi_d[z(n)] \equiv z(n)z^*(n) - \frac{1}{2}[z(n - 1)z^*(n + 1) + z(n + 1)z^*(n - 1)]. \quad (3.23)
\]

Two examples of the output of TK operator applied on rectangular and RRC
pulses, respectively, are shown in Fig. 3.12. For the rectangular pulse shape,

---

\textsuperscript{9}By high resolution (or superresolution), here we understand the ability to separate closely
spaced paths or echos.
the peak is clearly distinguishable after applying the TK operator. The property is preserved in closely spaced multipath channels. For the RRC case, we notice smaller sidelobes in the correlation function after the TK processing. However, the TK behavior in the presence of RRC pulses is not so intuitive as in the presence of rectangular pulses and we mainly relied on the simulation results in studying the behavior in closely spaced-path channels with bandwidth limitation.

![Figure 3.12: TK applied on the ACF of rectangular and RRC waveforms.](image)

The principle and the properties of the TK delay estimation algorithm are described in detail in [P4], and partially also in [P5]–[P7]. The TK algorithm performance is very promising for CDMA delay estimation in terms of accuracy and complexity [P4], [P6]. However, its performance degrades in the presence of RRC pulse shaping compared to the ideal case with rectangular pulses [P4], [P6]. Improvements of TK-based solutions (such as combined deconvolution and non-linear processing) are challenging topics for further investigation.

The mentioned delay estimation algorithms have been presented in more detail in [P6], together with simulation results. The most promising techniques in the context of CDMA applications (from the point of view of delay estimation accuracy) are the POCS variants and the TK algorithm. For the design of Rake receivers, when low-complexity requirements are usually more stringent than the requirements of very high delay estimation accuracy, the pulse-subtraction methods may also be good alternatives. All the described techniques, when applied in the context of bandlimiting pulse shapes, such as the RRC waveforms, still suffer from rather significant performance degradation compared to the situation when rectangular pulses are used (i.e., no bandwidth limitation). Optimizing the bandlimiting pulse shape to increase the delay estimation accuracy is still an open problem.
Chapter 4

Applications

4.1 Practical Rake architectures with channel estimator

A baseband Rake receiver should operate with at least 2 samples per chip to allow accurate signal reconstruction via samples (no aliasing) [160] and to reduce the signal degradation due to non-ideal sampling [52], [77], [104]. For accurate code synchronization, usually more than 2 samples are needed at the receiver [142]. This can be achieved via oversampling or via interpolation.

4.1.1 Oversampling-based methods

The oversampling-based Rake architectures are widely used in theoretical and simulation models. However, the term of “oversampled Rake” appears quite seldom in the literature [171], [183] as such; usually an oversampling factor higher than 2 is specified in the simulations or theoretical analysis, without further details on the receiver architecture [74], [104], [123], [217].

4.1.2 Interpolation-based methods

An alternative to the oversampling-based receiver is to use a low sampling rate (e.g., 2 samples per chip), followed by interpolation, in order to achieve the desired accuracy of the code synchronization process [P2], [27], [52], [57], [180], [211]. The interpolation block can be used either directly at the output of the pulse-shape matched filter, at sample level [27], [52], [57], [211], or at the output of the despreading unit, at symbol level [P2], [180]. The advantage of the second approach is a reduced number of operations. We showed in [P2] that this architecture did not deteriorate the code synchronization process and even a low-order interpolation, such as second order Lagrange interpolation, could be used with good performance (both in terms of BER and delay estimation accuracy).
The oversampling-based and the proposed interpolation-based architectures for the Rake receiver are shown in Fig. 4.1. In this figure, \( s_{ref} \) is the reference signal at the receiver, i.e., the code sequence plus optional data removal (in DA/DD modes). For both architectures, the samples of the received signals are first demultiplexed (into \( N_s \) branches for oversampling architecture, and into 2 odd and 2 even sequences for the interpolation architecture), such that the multiplication with the reference signal is done at chip rate.

![Diagram of oversampling and interpolation-based Rake receiver architectures.](image)

Figure 4.1: Oversampling versus interpolation-based Rake receiver architectures.

In the interpolation-based architecture, the sampling clock is reduced to only 2 samples per chip, which decreases the complexity of the baseband matched filter [180]. On the other hand, by the additional interpolation stage, we ensure that enough accuracy for the delay estimates is obtained.

### 4.1.3 Complexity issues

The complexity of the proposed interpolation-based Rake architecture with incorporated delay tracking unit was analyzed in [180]. Here we illustrate with a short example the gain related to the number of operations (i.e., multiplications and additions) for the proposed interpolation-based receiver architecture. The complexity computations are divided into two parts: the baseband-matched-filter complexity and the delay-tracking-unit complexity. The channel coefficient estimation unit (e.g., PADD) is not incorporated, because the complexity of this part will be the same for the two architectures. However, both the interpolation blocks (for delay estimation and for channel coefficient estimation) shown in Fig. 4.1 (right plot) are included in the complexity computations. The interpolators are second order Lagrange interpolators (studied in [P2]); this choice provides the best tradeoff between the complexity and the accuracy of the delay estimates. The
Table 4.1: Complexity comparison between the interpolation-based and oversampling-based architectures

<table>
<thead>
<tr>
<th></th>
<th>Overs. ((N_s = 4))</th>
<th>Interp.</th>
<th>Ratio Overs/Int.</th>
</tr>
</thead>
<tbody>
<tr>
<td>(SF = 8)</td>
<td>baseband filter</td>
<td>194560</td>
<td>304640</td>
</tr>
<tr>
<td></td>
<td>delay est.</td>
<td>2566</td>
<td>20483</td>
</tr>
<tr>
<td>total</td>
<td>197126</td>
<td>325123</td>
<td>85777</td>
</tr>
<tr>
<td>(SF = 512)</td>
<td>baseband filter</td>
<td>194560</td>
<td>304640</td>
</tr>
<tr>
<td></td>
<td>delay est.</td>
<td>46</td>
<td>20483</td>
</tr>
<tr>
<td>total</td>
<td>194606</td>
<td>325123</td>
<td>81997</td>
</tr>
</tbody>
</table>

choice of the baseband-filter implementation is described in [180] (we used a low-complexity interpolated FIR filter as that proposed in [151]). The delay estimation part is a simple feedforward structure (as in Fig. 3.11) without any superresolution processing (the search for maximum peaks is done directly on the averaged envelope of the correlation function, with non-coherent averaging length \(N_{NC} = 10\)). The oversampling-based receiver operates at \(N_s = 4\) samples per chip, and the interpolation-based receiver at \(N_s = 2\) (the interpolation degree is 2, such that we have comparable delay estimation accuracy with the two architectures). The complexity figures\(^1\) are shown in Table 4.1 for two values of the spreading factor. We notice that less than half of the operations are needed for the interpolation-based architecture. The gain is even higher when the comparison is done against higher oversampling factors.

### 4.2 Mobile positioning

#### 4.2.1 Short overview of mobile positioning problem

Mobile positioning (or radiolocation problem) has gained considerable attention during the past decade, triggered, on one hand, by standardization requirements, and, on the other hand, by economic factors such as the potential of new added-value services to 3G networks (e.g., location-specific traffic information, efficient resource management, vehicle tracking and intelligent transportation systems, etc.). The radiolocation methods are traditionally divided into 3 categories:

\(^1\)Here, the number of additions and multiplications is used as a simple measure of the receiver complexity.
those based on the signal strength, those based on the angle of arrival (AOA) and those based on the time of arrival (TOA). Combinations between these basic techniques are also possible and widely encountered in practice. Comprehensive overviews on the existing positioning methods can be found in [33], [35], [99], [189].

The most encountered radiolocation methods in today’s CDMA systems are the TOA methods and their variants, such as the time difference of arrival (TDOA). The underlying principle of TOA and TDOA methods is that the distance measurements are obtained from the measurement of the propagation time between the mobile and 3 or more BSs (or satellites for GPS case). In asynchronous networks, such as WCDMA network, the TDOA methods are preferred to TOA methods, in order to avoid the need of a universal clock. By using 3 or more estimated distances, the mobile position is found as the solution of a hyperbolic set of equations [36]. The estimate of the distance between the transmitter and the receiver is accurate provided that the receiver is in LOS situation with respect to the transmitter. However, the LOS signal might be obstructed by stronger NLOS rays arriving within short delays (i.e., closely spaced-path problem) or it may be completely absent (e.g., indoor propagation).

The NLOS propagation (or, equivalently, the multipath propagation) is one of the major sources of error in mobile positioning applications. Other sources of error in radiolocation are the hearability problem (i.e., inability to receive the signal from at least 3 BSs with strong enough power) [99], the multiaccess interference [33] and the presence of Doppler shifts.\(^2\)

The main problem addressed in this thesis is the estimation of LOS with high accuracy, targeting at TOA/TDOA mobile positioning methods. The first arriving path is considered to be a LOS path, and situations when it is weaker than its following neighbor rays are also analyzed (e.g., in [P7] and [P8]). The related problems of detecting whether LOS is present or not and estimating LOS out of NLOS components (when LOS is completely absent) are much more difficult and there are not many viable solutions in the research literature at the moment. Some existing solutions for LOS detection are overviewed in Section 4.2.2, and, afterwards, Section 4.2.3 deals with our proposed solutions for the estimation of the first arriving path.

4.2.2 LOS detection

The first methods to detect whether LOS is present are based on the history of the range measurements (i.e., the measurements of the time of arrival) and have initially been proposed by Wylie & al. in [212], [213]. In a range-measurement solution, the distance measurements are assumed to be equal to the true distances\(^2\) 

\(^2\)The Doppler shift can be neglected in terrestrial communications, being very small; it represents a major problem in satellite communications.
plus some estimation noise, whose variance is usually assumed to be known. The measurement process is either modeled as a high-order polynomial filter [212] or as a random process with exponentially decaying correlation and variable mean, according to whether LOS was present or absent [213]. The current-position estimate can be derived from previous estimates (e.g., via regression) and, based on certain \textit{a priori} assumptions, it can be decided if we are in a LOS or NLOS situation. These two algorithms were reported to increase the accuracy of the location estimation, but they are based on the strong assumption that \textit{a priori} knowledge of the measurement-noise variance is available. The ideas of [212] were also developed in [192], [210], where a Kalman filter was applied instead of polynomial smoothing.

Another range-measurement solution is the residual weighting algorithm [39]. This algorithm is based on the assumption that the mobile is in connection with 4 or more BSs. Several position estimates are formed using subsets of 3 BSs at the time and the most likely subset (in the sense of MSE error) is kept for the final position estimate. In this algorithm, the NLOS cases are not explicitly detected, but rather NLOS error is reduced, as reported in the simulations of [39]. However, the assumption that more than 4 transmitter-receiver connections are available is very limiting for practical applications.

More recent solutions try to use statistical information of the fading paths and to decide whether the LOS component is present or not based on some statistical tests (e.g., a Rician distribution with high Rician factor is likely to show a LOS situation, while a Rayleigh distribution is more likely to show a NLOS situation) [117].

4.2.3 LOS estimation

When LOS situation exists, one of the main problems in estimating LOS is the presence of closely spaced paths, which are overlapping with the first arriving path, as explained in Section 3.3.2. The same algorithms presented in Section 3.3.4 may also be used for accurate LOS delay estimation in the presence of overlapping paths and two examples are discussed in [P7] and [P8] for WCDMA scenarios. From the point of view of mobile positioning, very high accuracy of LOS delay estimates is usually targeted, even at the expense of a slightly increased complexity. Therefore, the methods with high potential for CDMA mobile positioning applications seem to be POCS and TK solutions.

When the number of channel paths is not \textit{a priori} known or estimated, the estimation of LOS delay from the overlapped correlation function can be done via thresholding: first, a ‘correlation’ (or cost) function is built, and then we search for the peaks higher than a certain threshold. The so-called ‘correlation’ function may be the output of any of the DSP algorithms mentioned in Section 3.3.4 (e.g., a TK processing plus non-coherent averaging [P7]).
The choice of the threshold is therefore important and statistical properties of the signal can be employed for defining efficient adaptive thresholds. An algorithm for the choice of the threshold is introduced in [P7]. The threshold methods have recently been studied also in [88], [201], [202]. In [88], [201], and [202] the LOS estimation is re-formulated as a composite hypothesis testing problem and the threshold is derived via constant false-alarm rate assumption.
Chapter 5

Analytical performance in the presence of channel estimation errors

A typical measure of performance of any digital receiver is its average bit error probability. BEP is a global measure of performance in the sense that it incorporates the effect of all transmitter-receiver stages, such as modulation/demodulation, coding/decoding, spreading/despreading, channel distortions (fading, MAI, additive noise), etc. Due to the complexity of the overall system, especially when random parameters (such as the fading parameters) are to be taken into account, the derivation of BEP is usually not a trivial task. Alternatively, the simulation-based approaches are widely used for the system performance analysis [9], [11], [37], [51], [78], [96], [148], [161], [164], [193]. In the simulation-based approaches we talk about bit error ratios instead of BEP, since these approaches are basically counting the number of errors which occur within a pre-defined period of time. BER term is also used in test equipment measurements, etc.

The purpose of this chapter is to present and compare the main existing approaches for BEP computation of CDMA receivers in fading channels. The main emphasis is on the Rake receiver architecture without channel coding. We also discuss here how the different channel estimator errors might be incorporated in the BEP analysis. When the channel coding is also taken into consideration, exact error probabilities are much more difficult to be obtained and union bounds on BEP may be used instead [175], [195].

We remark that alternative measures of performance, when describing a channel estimator algorithm, are the bounds on mean and variance of the estimation errors, such as the Cramer Rao Bound. CRB serves as a lower bound for the variance of any unbiased estimator and some variants adapted to multipath fading channels can be found in the literature [64], [131]. However, BEP and BER
measures are still the most powerful performance measures from the point of view of the receiver design, being the most revealing about the global behavior of the system.

The main general methods of computing BEP in fading and non-fading channels for any type of receiver can be found in a concise form in Chapter 8 of [145]. In what follows we explain them in the context of fading multipath channels, MRC spread-spectrum receivers and in the presence of channel estimation errors. The underlying idea of the method proposed in [P9] for computing BEP in the presence of code synchronization errors is also briefly reviewed.

There are basically two main methods of BEP analysis: the first one is based on the Gaussian approximation of the various sources of interference over the channel, as described in Section 5.1. The second method is based on the expression of the bit error probability of a quadratic receiver. The quadratic-receiver concept was initially introduced by Bello [18]. Later on, Barrett gave the exact expression for BEP of a quadratic receiver with binary modulation, under the assumption of Rayleigh fading channels [16]. Since then, the Barrett-based approach has gained considerable attention in CDMA applications [64], [91], [97], [121]. The analysis based on the quadratic-receiver model is explained in Section 5.2.

5.1 Gaussian approximations

The most encountered analytical method for the BEP calculation of digital receivers is the so-called standard Gaussian approximation (SGA), where the overall bit error probability $P_{a, SGA}$ is obtained as a function of the SNR $\gamma$ at the output of the receiver [24], [136], [145], [160] when the all the interference sources are modeled as additive white Gaussian noise:

$$P_{a, SGA} = \int_0^\infty P_b(\gamma)p_\gamma(\gamma)d\gamma,$$

where $P_b(\gamma)$ is the bit error probability conditional to the SNR, and it usually depends only on the modulation/detection scheme, and $p_\gamma(\gamma)$ is the PDF of SNR. For example, for BPSK modulation, BEP is [160] $P_b(\gamma) = Q(\sqrt{2\gamma})$, where $Q(\cdot)$ is the standard Gaussian Q-function $Q(x) \triangleq \int_0^\infty \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{\vartheta^2}{2}\right)d\vartheta$. Expressions for other modulations types can be found in [160].

Therefore, the underlying assumption of the SGA method is that all the interference sources (such as IPI, ICI, ISI, MAI) can be modeled as Gaussian random variables (according to the central limit theorem). Their variances are added to the standard additive Gaussian noise variance and they are reflected in the overall received signal-to-noise ratio (here, SNR is actually used with the meaning of 'signal to noise plus interference ratio').
When the PDF of the SNR is replaced by its dual parameter, i.e., the moment generating function\(^1\), the SGA approach is also referred to as \textit{MGF-based approach} \cite{8,175}. For a low number of interfering users, SGA proved out to give too optimistic results, and improved Gaussian methods were suggested \cite{120,146,215}. The improvement consists mainly of working with a conditional variance of the interference, i.e., computing the corresponding conditional SNR and error probabilities, before averaging over the distribution of the conditional variance \cite{120}.

Under several simplifying assumptions, such as ideal channel estimation, no coding, single user and MRC combining scheme, the SNR at the receiver output is a quadratic form in Gaussian variables (with zero or non-zero means, according to the fading type), whose probability distribution function and moment generating function are well-known (both for Rayleigh and Rician fading channels) \cite{7,8,160,175,196} and the integral expression of Eq. (5.1) can be expressed as a finite sum \cite{160,199}. For example, for an MRC receiver\(^2\) with \(L\) Rayleigh fading paths of distinct average powers \(P_l, l = 1, \ldots, L\), BPSK modulation, equal bit energies \(E_b\) for all transmitted bits, single-user case, uncorrelated distant paths and ideal channel estimation, the closed form expression of BEP is \cite{160}:

\[
P_{a,SGA} = \frac{1}{2} \sum_{l=1}^{L} \left( \prod_{l_1=1 \atop l_1 \neq l}^{L} \frac{P_l}{P_{l_1} - F_l} \right) \left( 1 - \sqrt{\frac{P_l}{N_0/E_b + F_l}} \right)
\]

(5.2)

Similar derivations may be applied for other combining schemes (e.g., equal gain combining, selection diversity, etc.) and for other modulation types.

However, when the channel estimation errors have to be taken into account, the analysis becomes much more tedious. In the presence of channel estimation errors, two main challenges appear when one desires to use the Gaussian approximation of Eq. (5.1). The first problem is the highly non-linear dependence of \(P_b(\cdot)\) on SNR. The second problem refers to the derivation of the particular distribution of SNR under imperfect channel estimation conditions (in the presence of channel estimation errors, SNR is no longer a quadratic form in Gaussian variables \cite{P9}).

For an accurate analytical analysis, the best known approach in current research community is the unified approach of Simon & Alouini, based on the alternative representation of Q-functions \cite{174,175}. This approach is briefly described in Section 5.1.1. Other approaches are based on many simplifying assumptions, such as assuming very slowly fading channels (or even constant within

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\(^1\)We recall that the MGF is the inverse Fourier transform of the PDF, and by definition, the MGF \(\Upsilon_x(\cdot)\) of a random variable \(x\) is equal to \(\Upsilon_x(\zeta) = E(e^{j\zeta x})\). MGF is also called the \textit{characteristic function}.

\(^2\)The SNR at the output of an MRC receiver with uncorrelated branches is the sum of SNRs of individual branches or paths \cite{160,175}.
the observation interval) [157], [188] or focusing only on single path scenario [14].

An alternative to the fully analytical derivation is the semi-analytical approximation, where the PDF of the SNR is computed via histogram methods and the integral is replaced by a finite sum. In Section 5.1.2, the author explains the use of Gaussian approximation in the context of imperfect delay estimation, fast Rician fading channels, MRC receiver and multiple access interference. A semi-analytical approach is used. This approach was introduced in [P9]. Because the variance of all the interfering processes (such as IPI, ICI, ISI and MAI) was conditioned to the fading and to the synchronization errors before taking the average over all possible fading process realizations, the approach in [P9] can be seen as belonging to the class of improved GA techniques.

5.1.1 Analytical methods based on the alternative representation of Q-functions

The main idea in the Simon & Alouini approach [175] is to replace the indefinite \(^3\) and infinite integration limits in the standard Q-functions (such as Gaussian Q-function or Marcum Q-function) with finite integration limits. For example, the Gaussian Q-function may be also re-written as [174], [175]

\[
Q(x) = \frac{1}{\pi} \int_{0}^{\pi/2} \exp \left( -\frac{x^2}{2\sin^2 \theta} \right) d\theta, \quad x \geq 0
\] (5.3)

This approach eases considerably the BEP computation (e.g., it is more suitable for numerical implementation and it allows to interchange the order of integration in analytical derivations). This method offer a wealth of new possibilities for finding closed-form expressions for average error probabilities, even if the fading statistics are neither Rayleigh, nor Rician. The unified approach of Simon & Alouini has widely been applied for CDMA signals with ideal Rake reception and various fading distributions [6], [7], [8], [174], [175], [176]. However, this method has not been applied yet (to the author’s knowledge) to non-ideal Rake reception, where channel estimation errors should also been taken into account.

5.1.2 Semi-analytical fast approach

A method to incorporate in the Gaussian approximation the delay estimation errors and the effects of tap spacing and pulse shaping was introduced in [P9]. The main assumptions are that the receiver is an MRC, that the spreading codes have ideal cross-correlation and auto-correlation properties, and that the channel coefficients are uncorrelated and with Rician fading amplitudes. The extension to the correlated fading was presented in [133]. The underlying idea is that the SNR

\(^3\)By indefinite limit we mean that it depends on some unknown variable.
at the output of MRC is no longer a quadratic form in Gaussian variables if the
delay errors are present, but rather a ratio of sums of bilinear forms in Gaussian
variables. Indeed, it was shown in [P9] that the SNR at the output of the Rake receiver
for the \( u \)-th user can be written as

\[
\gamma^{(n)}_u = \frac{E_{b_u}}{I^{(n)}_u} \left| B^{(0,n)}_{0,u,u} \right|^2 ,
\]

(5.4)

where \( I^{(n)}_u \) is the sum of IPI, ICI, ISI, MAI and AWGN interferences, given by:

\[
I^{(n)}_u = E_{b_u} S_{F_u} \sum_{v \neq u} \sum_{m = -\infty}^{+\infty} \left| B^{(m,n)}_{k,v,u} \right|^2 + N_0 (c^{(n)}_u)^H R^{(n)}_{1u} c^{(n)}_u ,
\]

(5.5)

and \( B^{(m,n)}_{k,v,u} \) is a bilinear form in complex Gaussian variables, depending on the
channel complex coefficient vectors:

\[
B^{(m,n)}_{k,v,u} = (c^{(u+m)}_v)^H R^{(m,n)}_{k,v,u} c^{(n)}_u .
\]

(5.6)

Above, \( R^{(m,n)}_{k,v,u} \) and \( R^{(n)}_{1u} \) are the pulse shaping correlation matrices, with elements:

\[
\rho^{(m,n)}_{R_{k,v,u}} (l, l_1) = R \left( (m + n) T_u - n T_v + k T_c + (\tau^{(n+m)}_{l_1,u} - \tau^{(n)}_{l,u}) T_s \right) ,
\]

(5.7)

and, respectively:

\[
\rho^{(n)}_{R^{(n)}_{1u}} (l, l_1) = R_1 \left( \tau^{(n)}_{l_1,u} - \tau^{(n)}_{l,u} \right) , \quad l, l_1 = 1, \ldots, L ,
\]

(5.8)

and \( R_1 (\cdot) \) is the auto-correlation function of the reference-code pulse shape

\[
R_1 (\tau) = \frac{1}{S_{F_u} N_s} \sum_{i = n S_{F_u} N_s}^{(n+1) S_{F_u} N_s} g_1 (iT_s + \tau T_s) g_1 (iT_s) dt.
\]

(5.9)

\[\text{A bilinear form in the Gaussian vectorial variables } x, x_1 \in \mathbb{C}^L \times 1 \text{ is an expression of the type } x^H B x_1, \text{ where } B \in \mathbb{C}^{L \times L} \text{ is the bilinear form matrix. When } x = x_1, \text{ the bilinear form becomes a quadratic form.}\]

\[\text{The channel complex coefficient vectors are denoted by } \xi \text{ in [P9].}\]
The advantage of having the SNR expression in the form of Eq. (5.4) comes from the fact that the individual effect of each of the interference terms can be seen separately from the others, which allows a better insight of the receiver behavior (e.g., study the PDF of the interference terms under various fading scenarios). Semi-analytical performance curves and the comparison with simulation results can be found in [P9].

Although the Gaussian approximation is a powerful tool for the receiver performance analysis, its main drawback comes from the fact that general and unified formulas are very difficult to be obtained (e.g., covering several types of diversity combining, including all types of estimation errors, etc.).

5.2 Quadratic-receiver model

Under the assumption that the number of Rake fingers is equal to the number of channel paths the first user’s MRC output (see Eq. (2.31)) can be also re-written as a quadratic form [91], [121], [138]:

\[ y^{(n)} = (x^{(n)})^H Q x^{(n)}, \quad (5.10) \]

where \( x^{(n)} \) is the random vector of estimated channel coefficients and despreading outputs

\[ x^{(n)} = \begin{bmatrix} \hat{c}^{(n)} \end{bmatrix} \]

and \( Q \) is the \( 2L \times 2L \) matrix

\[ Q = \frac{1}{2} \begin{bmatrix} O_L & I_L \\ I_L & O_L \end{bmatrix}. \quad (5.12) \]

Above, \( I_L \) is the identity matrix of dimension \( L \times L \), and \( O_L \) is an all-zero matrix of dimension \( L \times L \). It follows from Eq. (5.10) that MRC receiver is a particular case of the more general quadratic receiver [16], [18].

Under the assumption of Rician fading channels, the vector \( x^{(n)} \) is a vector of complex Gaussian distributed variables having the means \( \mu_i^{(n)} \), \( i = 1, 2, \ldots, 2L \). The means of the Gaussian distributed variables are zero only when the channel is Rayleigh fading. The moment generating function \( \Upsilon^{(n)}_{MGF} (\zeta) \) of the quadratic form \( y^{(n)} \) in complex Gaussian variables is well-known since Turin [196]:

\[ \Upsilon^{(n)}_{MGF} (\zeta) = \prod_{i=1}^{2L} \exp \left( \frac{j \lambda_i^{(n)} |\mu_i^{(n)}|^2}{1 - j \lambda_i^{(n)} \zeta} \right) \frac{1}{1 - j \lambda_i^{(n)} \zeta} \quad (5.13) \]

\(^6\)The model can be easily extended to \( L_r \neq L \), either by taking only a subset of the set of channel paths, if \( L_r < L \), or by padding with zeros the channel coefficient vector.
where $\lambda_i^{(n)}$, $i = 1, 2, \ldots, 2L$ are the eigenvectors of the matrix $\Sigma_{xx}^{(n)} Q$, with:

$$\Sigma_{xx}^{(n)} = \mathbb{E} \left( (x^{(n)})^H x^{(n)} \right) \triangleq \begin{pmatrix} \Sigma_{c\,c}^{(n)} & \Sigma_{c\,z}^{(n)} \\ \Sigma_{z\,c}^{(n)} & \Sigma_{z\,z}^{(n)} \end{pmatrix}. \tag{5.14}$$

The matrices of Eq. (5.14) can be further developed using the matrix model of Eq. (2.23) and basic matrix operations. The channel estimation errors have already been incorporated in the expression of the channel coefficients vector $\hat{c}^{(n)}$, and in the despreading matrix $\hat{S}$. If the channel coefficient estimation errors are assumed to be Gaussian distributed of zero-mean, then the presence of an error in the channel complex coefficients estimates is signalled only in the sub-matrix $\Sigma_{c\,c}^{(n)}$ of Eq. (5.14). The delay estimation errors affect the despreading matrix $\hat{S}$, and hence they influence 3 sub-matrices $\Sigma_{c\,z}$, $\Sigma_{z\,c}$ and $\Sigma_{z\,z}$.

The probability distribution function $p_y(y^{(n)})$ is the Fourier transform of the MGF and it is generally difficult to obtain in closed form because of the complicated nature of the exponential factor in Eq. (5.13). However, in the special case when the Gaussian variables have zero means (i.e., Rayleigh fading channels), an exact expression of the PDF $p_y(y^{(n)})$ can be obtained via the residue theorem [97], [160], [196].

Based on the MGF of the quadratic form in Gaussian variables, a general formula for the binary error rate in fast Rayleigh fading channels was derived in [16]. Barrett showed [16] that, if $x^{(n)}$ is Gaussian distributed of zero mean, the bit error probability $P_b$ conditional to the transmitted bit (and under the hypothesis of distinct eigenvalues) is:

$$\left( P_b | y^{(n)} \right) = \sum_{i=1}^{2L} \prod_{j=1}^{2L} \frac{1}{\lambda_i^{(n)}}, \tag{5.15}$$

The formula (5.15) is valid for any binary modulation, and can be also applied to QPSK modulations [122], [160]. Furthermore, approximations for higher-order modulations based on Eq. (5.15) are also possible (e.g., MFSK [16]). To avoid the zeros at the denominator, we need distinct eigenvalues. For multiuser and multipath interferences, it is highly unlikely to obtain equal eigenvalues. In the numerical analysis, the situation with equal eigenvalues can be treated by introducing small variations which should not affect BEP in a significant way [121].

To compute the average bit error probability $P_{a\,Barrett}$, one has to compute the BEP for each of the possible $M^{N_{sym}}$ combinations ($M$ being the modulation alphabet size and $N_{sym}$ the total number of transmitted symbols), and then, one has to average over all these combinations. In practice, under the assumption of BPSK and QPSK modulation, good estimates can be also obtained via the
simplified formula:

\[ P_{a,\text{Barrett}} = \frac{1}{N_{\text{sym}} \sum_{n=1}^{N_{\text{sym}}}} (P_b|b^{(n)}) \], \quad (5.16) \]

where \( N_{\text{sym}} \) is a sufficiently high number of symbols.

Barrett-based BEP analysis has found several applications in the context of CDMA receivers, Rayleigh fading channels and ideal channel estimation [26], [64], [91], [97], [121]. The impact of channel estimation errors via the quadratic-receiver model (and especially the impact of the channel coefficient estimation errors) was also addressed by few authors in the context of Rayleigh fading channels [64], [65], [91].

Two examples of applying Barrett-based BEP analysis in the context of Rake receivers with channel estimation errors and Rayleigh fading channels are shown in Figs. 5.1 and 5.2. Fig. 5.1 compares the theoretical Barrett-based BEP with the COSSAP simulation results\(^7\) in the presence of imperfect channel coefficient estimates. The channel coefficient estimation errors are assumed to be Gaussian distributed of zero means and equal variances for all paths. The channel has 1 or 2 Rayleigh fading paths, the spreading factor is \( S_F = 128 \), the pulse shape waveform is an RRC waveform with rolloff 0.22. The plots are for single-user case (because the main idea is to illustrate the effect of channel coefficient estimation errors under otherwise ideal conditions). The theoretical BEP agrees well with the simulation results.

Fig. 5.2 shows an example of BEP analysis in the presence of code synchronization errors, via Barrett-analysis and via the SNR-based approach proposed in [P9]. All four channel paths are assumed to have the same constant delay error, given in the legend of the figure. The results obtained with the two different methods are matching perfectly at low delay estimation error and they also agree pretty well when the synchronization errors are higher than \( 0.5 \ T_c \). The Barrett-based analysis is taking into account the imperfect code properties, and therefore tend to be slightly more pessimistical than SNR-based approach.

The advantages of BEP analysis based on the quadratic receiver are multiple, and this analysis seems to be one of the most powerful and accurate analytical tools existing nowadays to characterize the receiver performance over Rayleigh fading channels. Despite the fact that the formulation of (5.14) is not very intuitive, it has the advantage that it incorporates the channel estimation errors, and, basically, any error distribution might be used in this context (i.e., the formula is valid for any channel estimation algorithm). Extensions to multiple sensors (i.e., transmit and receive antenna diversity) have also been studied for Rayleigh fading channels [26], [92].

\(^7\)A downlink WCDMA simulation model was built with COSSAP simulation tool, according to the 3G current specifications [1]. More detailed description of the simulation environment can be found in [P1].
Figure 5.1: BEP in the presence of channel coefficient estimation errors; one and two Rayleigh fading paths.

Figure 5.2: BEP in the presence of delay estimation errors; 4 Rayleigh fading paths.
On the other hand, Barrett-based approach suffers also from few drawbacks. The main one is that the closed form expression of Eq. (5.15) is valid only for Rayleigh fading channels (or zero-mean complex Gaussian vectors $\mathbf{x}^{(n)}$). For Rician fading channels with non-zero mean Gaussian variables, no trivial expression for BEP does exist and saddle point approximations should be used in computing the integrals that define the bit error probability [116], [136]. This makes the Barrett-based approach quite impractical for the analytical study of the performance under generic Rician fading channels. Another drawback comes from the rather heavy computational effort involved in computing the covariance matrices of Eq. (5.14) and the eigenvalue decomposition. The covariance matrices $\Sigma_{c,c}^{(n)}$, $\Sigma_{a,c}^{(n)}$ and $\Sigma_{a,z}^{(n)}$ depend both on the spreading sequences and the pulse shaping factors. It means that for accurate BEP expressions, an average over all possible set of spreading sequences is usually desired, which increases the computational time. When the code synchronization errors are also taken into account, this computational time is further increased, because the average over all possible delay errors is also needed.
Chapter 6

Summary of publications

The second part of the thesis is based on nine publications. None of these publications has been used as part of any other dissertation.

The main topic considered in these publications is the development of new delay estimation algorithms and theoretical analysis of the fading multipath channels, in the presence of spread spectrum signals, additive white Gaussian noise and, possibly, multiple access interference. The simulations and theoretical models focus on the downlink WCDMA receiver, with the main emphasis on Rake receiver type structures and mobile positioning applications. However, many of the algorithms described here could find application in more general contexts as well, such as in uplink scenarios and advanced multiuser detection structures.

6.1 Description of the main results

Publication [P1] deals with the channel estimation problem in a Rake receiver. Publication [P2] illustrates a practical application of the channel estimation algorithms in the context of a reduced complexity Rake receiver based on interpolation. Publications [P3] to [P8] present different delay estimation algorithms for closely spaced path scenarios. Publications [P3] to [P6] focus on the Rake receiver applications. The comparison of different delay estimation algorithms in terms of performance and complexity, together with the steps to be considered for practical receiver design, are given in the publication [P6]. Publications [P7] and [P8] deal with LOS estimation algorithms for mobile positioning. Two of the most promising solutions in the context of LOS estimation in CDMA systems with RRC pulse shaping, namely TK algorithm with adaptive threshold setting [P7] and POCS algorithm with additional constraints [P8], are studied in the context of downlink WCDMA mobile positioning. The Rayleigh fading situation was considered there as a worst-case scenario. Publication [P9] focuses on the theoretical analysis of the Rake receiver performance in the presence of delay estimation errors.
In [P1] we analyzed the robustness of a Rake receiver in the presence of delay estimation errors, assuming that pilot-aided approaches were used for the estimation of the amplitudes and phases of the fading paths. The delay and the complex coefficient estimation tasks were addressed independently and we studied and compared several combinations of different solutions for each of these two tasks. The original feedforward architecture for the delay estimation part was proposed in [132], but it was studied there only under the assumption of known channel coefficients. The advantage of splitting the two channel estimation tasks comes from the reduced complexity and the easier implementation in a Rake receiver compared to the joint maximum-likelihood channel estimation. We showed in [P1] that, when the pilots were time-multiplexed with the data symbols, the best results were obtained with a combined PADD coefficient estimation and an NDA or an DD delay estimation scheme.

In publication [P2] we introduced a feedforward interpolation-based delay tracking unit for a Rake receiver as a low-complexity alternative to the receivers based on oversampling. We presented the detailed architecture of the delay tracking unit and we discussed and compared different interpolation methods. The comparison with the oversampling-based Rake receiver and with the receiver operating at one sample per chip was also shown. We note that more details about the complexity of the overall Rake receiver, including the baseband filtering, can be found in [180].

In [P3] an improved pulse subtraction method was proposed for the delay estimation in closely spaced multipath scenarios and square root raised cosine pulse shapes. The method was implemented in a feedforward manner. It was shown that, for RRC pulses, this method was significantly better than the traditional envelope tracking with subtraction methods.

An efficient technique for asynchronous CDMA multipath delay estimation was presented in [P4]. This technique is based on the complex Teager-Kaiser operator and has subchip resolution capability. Its performance was compared with the performance of other super-resolution algorithms, adapted for asynchronous systems with long codes. We also analyzed in [P4] the impact of the pulse shape waveform on the performance of the algorithm. The ideas presented here can be applied in multicell downlink WCDMA scenarios, where multiple BSs transmit asynchronously, as well as in an uplink CDMA scenario, where the users’ signals are not synchronized. We mention that we first proposed the delay estimation algorithm based on the real TK operator in [70], for GPS receivers with short codes and rectangular pulse shaping. The TK delay estimation algorithm was also studied for GPS applications in [69].

In [P5] a MUSIC-based delay estimation algorithm for CDMA systems with long codes was presented and its performance was compared with that of the TK delay estimation algorithm. For the simulation results, we used a single cell synchronous downlink WCDMA scenario with closely spaced paths. We showed that
in the presence of RRC pulse shaping the performance of both algorithms was worse than in the presence of rectangular pulse shaping, and that there were only few cases when TK algorithm was outperformed by the MUSIC-based algorithm.

In [P6] we provided a comprehensive review of the main existing delay estimation techniques which are able to cope with multipath propagation in CDMA systems. Particular attention was devoted to the closely spaced path situations. Some of the techniques presented here have previously been used in the context of CDMA systems; some others were proposed by the authors for CDMA applications. We discussed both the closed-loop and open-loop implementations, with their advantages and drawbacks. The closed-loop algorithms, such as the DLLs and the EKF were usually found to be rather complex for CDMA mobile receivers. Simulation results were provided, together with a unified theoretical framework. The most promising methods for CDMA delay estimation in terms of accuracy were found to be the TK and POCS algorithms. The simulations were carried out for asynchronous WCDMA cells with synchronous downlink users.

In [P7] we proposed an adaptive threshold technique for the link-level estimation of the first arriving path in a CDMA multipath environment. The adaptive threshold technique was used in conjunction with TK delay estimation, but the main principles can be further applied with any multipath delay estimation algorithm.

In [P8], we introduced a new deconvolution algorithm, based on the projection-onto-convex-sets idea, for LOS estimation in WCDMA systems, in the presence of severe multipath and multicell interference. By incorporating additional constraints in the iterative steps of the POCS algorithm, we showed that the robustness as regards noise and interference can be significantly increased.

In [P9] we introduced a simple and accurate semi-analytical method for computing the bit-error probability of a Rake receiver with code synchronization errors and Rician uncorrelated fading paths. The code synchronization errors were quantized in terms of bilinear forms of complex Gaussian variables. The theoretical results were validated by link-level simulations with WCDMA long codes with high spreading factors. The simulation tools were COSSAP and Matlab. We note that we presented an extension of this work to correlated fading paths in [133].

6.2 Author’s contribution to the publications

The research work for this thesis was carried out at the Institute of Communications Engineering (formerly the Telecommunications Laboratory) as part of the wider research projects “Digital and Analogue Techniques in Flexible Receivers”, “Advanced Transceiver Architectures and Implementations for Wireless Communications”, and “Advanced Techniques for Mobile Positioning”. During the work, the author has been a member of an active research group, involved in study-
and developing DSP algorithms for CDMA receivers and mobile positioning. Many of the ideas have originated in discussions within the group and some of the simulation models, built in Matlab, COSSAP, or System Studio, have been designed in cooperation with the co-authors. Therefore, the author’s contribution cannot be separated completely from the contributions of the co-authors.

However, the author’s contribution to all of the publications included in this thesis has been essential in that she has carried out the main theoretical derivations, developed new algorithms, performed most of the simulations, and prepared the manuscripts. Naturally, the co-authors contributed to the final appearance of each article. The main contribution of the author to the publications can be summarized as follows:

In publication [P1] the author built the new delay estimation schemes, elaborated the theoretical framework, implemented the channel estimation algorithms in COSSAP, carried out all the simulations, and wrote the manuscript. The original idea of using feedforward structures for channel estimators belongs to Professor Markku Renfors. The PADD channel coefficient estimation algorithm in publication [P1] was derived in close cooperation with the co-authors. The author received also some help from other members of the Institute of Communications Engineering in building the Rayleigh fading channel simulator in COSSAP environment.

The idea of using a feedforward interpolation-based architecture in publication [P2] belongs to Professor Markku Renfors. The architecture of the delay tracking unit was designed with the help of the co-authors. The author proposed and analyzed the performance of different interpolators, both in time domain and frequency domain, built the COSSAP and Matlab models and carried out the simulations. The calculus of the Rake delay tracking unit complexity was the result of the joint work of the co-authors.

In publication [P3], the author proposed the improved pulse subtraction algorithm, developed both the theoretical and simulation parts of the algorithm, and prepared the manuscript. She also studied the choice of the best pulse used in the search algorithm. Originally, Professor Markku Renfors introduced the problem of closely spaced multipath components to the author and gave the idea of using a pulse subtraction approach.

In [P4], the author is the co-developer of the multipath delay estimation algorithm based on the nonlinear complex Teager-Kaiser operator. The author proposed the use of the Teager-Kaiser delay estimation algorithm in the context of asynchronous CDMA environments. The author also built the simulation model, performed all the simulations, and wrote more than half of the manuscript. The theoretical model was built in close cooperation with co-authors. Originally, the idea of TK processing was introduced to the author by Dr. Ridha Hamila.

In [P5] the author designed the MUSIC-based algorithm for the long CDMA codes and developed the theoretical framework for the MUSIC estimator. The au-
thor also carried out most of the Matlab simulations and prepared the manuscript.

In [P6], the author re-formulated the problem of the CDMA delay estimation in terms of a deconvolution problem and proposed different signal processing algorithms as solutions for this problem, namely the least-squares and constraint least-squares, the quadratic programming, and the MUSIC algorithms. The author also showed the connection between different ML-based delay estimation algorithms, namely the pulse subtraction, the improved pulse subtraction, and the MEDLL. She also compared and analyzed the different variants of the delay locked loops and emphasized their drawbacks for CDMA applications. All the delay estimation algorithms were re-defined by the author in a unified framework, valid for any type of pulse shaping, and for both closely spaced and distant path scenarios. The EKF algorithm was derived and implemented jointly with the co-authors. The PS and TK algorithms were based on previous published ideas in co-operation with the co-authors. The author also received useful suggestions from the co-authors related to the parameter optimization and to the theoretical derivations. She implemented most of the simulations and wrote the manuscript.

In [P7], the author developed the theoretical framework, proposed the empirical threshold, and performed most of the simulations. The collaboration with the co-authors helped in refining the parameters of the algorithm.

In [P8], the author proposed the new deconvolution approach for LOS estimation. She also carried out the theoretical derivations, implemented the algorithm, and wrote the manuscript. The problem of the closely spaced multipaths and the implications in the context of LOS estimation were originally introduced to the author by Professor Markku Renfors.

The author developed the semi-analytical algorithm of publication [P9], performed the simulations, and wrote the manuscript. The starting point for the theoretical model was the result of several fruitful discussions with Professor Markku Renfors and Professor Markku Juntti (during a research visit at the University of Oulu). The challenges related to the impact of code synchronization errors on Rake receiver performance, both in uncorrelated and correlated fading scenarios, were introduced to the author by Professor Markku Renfors.
Chapter 7

Conclusions

This thesis covered several channel estimation issues related to CDMA communications over multipath fading channels. The problematics, the motivation, the prior art, and the parallel work were introduced in Chapter 1.

After presenting the channel model and the main channel parameters in Chapter 2, we included in Chapter 3 an overview of the main techniques for the decoupled channel coefficient estimation and delay estimation. The focus was on the low-complexity Rake receiver structures, with decoupled channel estimation and decoding blocks. We showed that a PADD estimation algorithm is a good option when a burst-mode pilot channel is available. We also discussed the choice of the fine channel coefficient estimation filter and we showed that an MA filter with fixed coefficients and optimally chosen adaptive length has a performance close to that of a Wiener filter and has the advantage of lower complexity.

For the delay estimation part we proposed feedforward architectures with an incorporated unit for the estimation of multipath delays with subchip accuracy. As discussed in Section 3.3.3 and illustrated with more examples in [P6], the feedforward architectures have the advantage of simplicity and better performance compared to the traditional feedback delay estimation structures. Moreover, the feedback loops such as DLL with IC, DLL with IM, and EKF may be used up to a certain extent to solve the problem of closely spaced paths, but they are usually quite complex and require some a priori information which is not easy to obtain [P6]. Several feedforward solutions were introduced to deal with the closely spaced path problem. The lowest-complexity solutions, among those proposed in the thesis, are the PS and TK algorithms [P3]–[P6]. From the point of view of Rake receiver design, these solutions seem to be the best choice, as explained in [P6]. From the point of view of positioning applications and possibly in the context of more advanced receiver architectures that are more sensitive to delay estimation errors, the most promising algorithms are TK [P4], [P6] and POCS [P6], [P8]. The POCS algorithm has the best performance under various pulse shapes and with closely spaced as well as distant paths; however, its complexity is usually
quite high [P6]. However, under certain environments and conditions, such as low delay spreads and perfect power control, the complexity of POCS can be significantly reduced. For the generic case, when no a priori information is available, the TK solution gives the best tradeoff between performance and complexity. We also emphasized the need for even more powerful delay estimation schemes when RRC pulse shaping is used to limit the signal bandwidth. The combination of the deconvolution approaches with an TK operator could be one solution for dealing with RRC pulse shapes and it remains a topic of future research.

Chapter 4 was dedicated to two CDMA applications where channel estimates are needed. The first one is the well-known Rake receiver, the building block of all present-day receivers. A good tradeoff between the code synchronization accuracy and reduced implementation complexity was found in a new interpolation-based architecture, where a low-order polynomial interpolation stage is applied in the narrowband domain to increase the accuracy of the multipath delay estimates. The second application referred to the mobile positioning in wireless networks. The solutions proposed for solving the closely spaced path problem were found to be very useful in the context of mobile location, where the LOS component may suffer from strong interference from other NLOS components. A new threshold for LOS delay estimation was also proposed [P7]. Due to its empirical derivation, it is likely that slightly different threshold parameters may be the optimum choices for other delay estimation schemes. However, the same idea behind deriving the threshold for LOS estimation is likely to give good results.

Chapter 5 presented the main methods to compute the BEP of a CDMA receiver in the presence of multipath fading channels and discussed the effect of channel estimation errors on the algorithm performance. Also, a new method for incorporating the multipath delay estimation errors in the GA when computing BEP was presented. It was shown that the semi-analytical approach for BEP derivation, proposed in [P9], represents a fast and efficient solution for studying the Rake receiver performance in the presence of code synchronization errors. Nevertheless, when the channel coefficient estimation errors are also present or when the combining scheme is other than MRC, Barrett-based analysis remains the most powerful tool offered in today’s literature. Open issues here are the extension (in an efficient way) of Barrett-based analysis to generic Rician or Nakagami channels, and the incorporation of channel coefficient estimation errors in the improved GA-based approach of [P9].

It remains as a challenging topic for future work to apply the algorithms presented in this thesis in the context of more advanced CDMA receiver structures (such as linear equalizers, non-linear IC receivers, or MIMO structures) and to some other CDMA based systems, such as the future satellite positioning system Galileo and the future 4G communications systems. Also, there is still an open question about how to extend the presented theoretical methods to CDMA systems with coding and adaptive modulation schemes.
Bibliography


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Publications

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